

# Understanding Persistent ZLB: Theory and Assessment<sup>\*</sup>

Pablo Cuba-Borda<sup>†</sup>      Sanjay R. Singh<sup>§</sup>

November 2023

## Abstract

We develop a theoretical framework that rationalizes two hypotheses of long-lasting low-interest rate episodes: *deflationary-expectations-traps* and *secular stagnation* in a unified setting. These hypotheses differ in the sign of the theoretical correlation between inflation and output growth that they imply. Using the data from Japan over 1998:Q1-2019:Q4, we find that the data favor the expectations-trap hypothesis. The superior model fit of the expectations trap relies on its ability to generate the observed negative correlation between inflation and output growth.

Keywords: Expectations-driven trap, secular stagnation, zero lower bound.

JEL Classification: E31, E32, E52.

---

<sup>\*</sup>This paper replaces a previous version circulated with the title “Understanding Persistent Stagnation”. We thank Boragan Aruoba, Florin Bilbiie, Carlos Carvalho, Gauti Eggertsson, Pascal Michaillat, Jean-Baptiste Michau, Taisuke Nakata, Giovanni Nicolò, Sebastian Schmidt, Kevin Sheedy and participants at various presentations for helpful comments and suggestions. We are particularly grateful to Yasuo Hirose for generously sharing data and codes from his paper. Disclaimer: The views expressed in this article are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System.

<sup>†</sup>Division of International Finance, Federal Reserve Board, Washington DC ([pablo.a.cubaborda@frb.gov](mailto:pablo.a.cubaborda@frb.gov))

<sup>§</sup>Federal Reserve Bank of San Francisco and Department of Economics, University of California, Davis ([sjrsingh@ucdavis.edu](mailto:sjrsingh@ucdavis.edu)).

“I believe that for the euro area there is some risk of *Japanification*, but it is by no means a foregone conclusion.” — Mario Draghi (January, 2020).

## 1. INTRODUCTION

Since the global financial crisis of 2008-2009, concerns about prolonged near-zero interest rates and meager inflation became predominant across many advanced economies, most notably in Europe and the United States. Such concerns, dubbed as *Japanification*, relate to the decades-long stagnation of the Asian economy following the collapse of a real-estate bubble in the early 1990s. As a result, nominal interest declined to zero, and deflation emerged, leaving the central bank unable to fight recessions.<sup>1</sup>

Two predominant hypotheses rationalize interest rates near zero and inflation below the central bank’s target. The first hypothesis is that of a *deflationary-expectations-driven* liquidity trap whereby deflationary expectations become self-fulfilling in the presence of the zero lower bound (ZLB) constraint on short-term nominal interest rates (Benhabib, Schmitt-Grohé and Uribe, 2001, 2002). The second hypothesis is *secular stagnation* that entails a persistently negative natural interest rate constraining the central bank at the ZLB (Hansen, 1939; Summers, 2013; Eggertsson, Mehrotra and Robbins, 2019). Existing research has shown that these two hypotheses offer contrasting policy implications (Bilbiie, 2022; Nakata and Schmidt, 2021).

This paper builds a theoretical framework that rationalizes *deflationary-expectations-driven* liquidity trap and *secular stagnation* in a unified setting. We analytically show that the empirical correlation of output growth and inflation can be used to distinguish the two hypotheses. Secular stagnation, by policy rates permanently stuck at zero, generates a positive correlation between inflation and output growth. On the other hand, because of local indeterminacy, deflationary expectation-trap models can generate a negative correlation between inflation and output growth. In Japan, the correlation between inflation and output growth is negative. We find that the superior model fit of the expectations-trap model relies on its ability to generate this negative correlation.

We augment the textbook New Keynesian model with a bonds-in-utility specification

---

<sup>1</sup>Financial Times, “*Japanification: investors fear malaise is spreading globally*,” August 26, 2019. ASSA Annual Meeting Panel Session: “*Japanification, Secular Stagnation, and Fiscal and Monetary Policy Challenges*,” January 2020.

(Michaillat and Saez, 2021; Michau, 2018; Ono and Yamada, 2018). This modification breaks the tight connection between the natural interest rate and the discount factor, thus allowing for a permanently negative natural interest rate. Within this framework, we define three steady-state equilibria. A *targeted-inflation* steady state at which the central bank can meet its inflation target, output is at potential, and the nominal interest rate is positive. In addition, there are two liquidity trap steady states at which inflation is below the central bank's intended target. The level of output is below potential, and the nominal interest rate is at the ZLB. The focus of our analysis is to compare these two liquidity traps.

The liquidity trap steady states arise due to the ZLB constraint on short-term nominal interest rates. In one case, combined with long-run money non-neutrality, a shift in the agent's inflation expectations makes the liquidity trap equilibrium self-fulfilling. For this reason, we label it deflationary-expectations-driven liquidity trap or *expectations trap* for short.<sup>2</sup> Alternatively, when prices are rigid, the economy can settle in a liquidity trap because of a permanent decline in the natural interest rate. In this case, the liquidity trap arises because of a change in the economy's fundamentals and not because of a shift in expectations. For this reason, we label this situation as a *secular stagnation* steady state or *fundamentals-driven* liquidity trap. In the absence of discounting, the natural interest rate is constant and equal to the inverse of the household's discount factor, and the model cannot accommodate the *secular stagnation* hypothesis.

We use the Japanese experience from 1998 to 2019 as a laboratory to contrast the two hypotheses and offer the first quantitative assessment of expectation-driven liquidity traps versus secular stagnation. We use a New Keynesian framework to assess if a policymaker can use the data to discern the predominant hypothesis. Using Bayesian prediction pools, we estimate the probabilistic assessment of the relevant model (Geweke and Amisano, 2011; Del Negro, Hasengawa and Schorfheide, 2016). Our quantitative analysis offers two main findings. First, we find evidence that Japan is more likely to be in an expectations-driven liquidity trap. Second, considerable real-time uncertainty exists between secular stagnation and the expectations-driven trap models, especially during Japan's first decade near the ZLB.

---

<sup>2</sup>Benigno and Fornaro (2018), and Ravn and Sterk (2021) present complementary models of expectations trap where pessimism about growth rate or unemployment risk can push the economy to the liquidity trap. In this paper, our focus is solely on the deflationary expectations trap.

We find that equilibrium indeterminacy is central to tilting our quantitative assessment in favor of the expectations-trap hypothesis. This result emerges because the dynamic properties of the ZLB equilibrium differ across the two narratives. Under secular stagnation, the ZLB equilibrium exhibits locally determinate dynamics. In contrast, the expectation traps model features locally indeterminate dynamics around the ZLB steady state. Thus, the equilibrium dynamics are consistent with a multiplicity of stable paths. Because our quantitative analysis focuses on a long-lasting ZLB episode, equilibrium selection implies restrictions for the response of output growth and inflation to structural disturbances. Our full-information approach ensures that the data selects the best-fitting equilibrium. At the same time, our Bayesian procedure intrinsically penalizes the likelihood function for the presence of additional parameters in the expectations-trap equilibria (Schwarz, 1978; Lubik and Schorfheide, 2004).

What accounts for the better fit of the expectations-trap hypothesis in Japan? The negative correlation between output growth and inflation in Japanese data is a key empirical moment for equilibrium selection and model fit. The equilibrium dynamics around the secular stagnation steady state cannot deliver the observed negative correlation. With interest rates pegged at the ZLB, any shock that generates a persistent increase in the inflation rate lowers the real interest rate and increases consumption; therefore, output and inflation positively co-move. In contrast, local indeterminacy of the expectations-trap steady state implies that inflation can adjust in any direction. Our estimation procedure allows the data to pin down this response. Expectation traps can generate an unconditional correlation between inflation and output growth close to that observed in the data.

We further investigate our empirical results along three dimensions. First, we investigate the importance of non-fundamental i.i.d. shocks—known as sunspots—that emerge due to indeterminate model dynamics in the expectations-trap model. Our benchmark result indexes equilibrium multiplicity through the correlation between fundamental and sunspot shocks using the method of Bianchi and Nicolò (2021). We find that restricting the correlation between price-markup and sunspot shocks to non-positive values worsens the fit of the expectations-trap model and favors secular stagnation. Thus, using data to discipline equilibrium selection is central to our results. We augment our estimation of the expectations-trap model with inflation expectations data to further discipline equilibrium selection. The prediction pool analysis still

favors the estimated expectations-trap model over secular stagnation. Second, we check whether departures from rational expectations modeling enhance the ability of the secular stagnation model to fit the data. We find that of the three departures we considered, cognitive discounting (Gabaix, 2020) helps improve the model fit for the secular stagnation model. However, it still falls short of the model fit implied by the expectations-trap hypothesis. Third, we verify that the expectations-trap hypothesis outperforms secular stagnation in an estimated medium-scale new Keynesian model. The correlation of markup shocks and sunspot shocks remains central to the model fit despite added complexity. This exercise implies that our analytical insights carry over to a wide class of models commonly used for policy analysis.

**Relation to the literature.** Our work complements the recent analyses of Michailat and Saez (2021), Michau (2018), and Ono and Yamada (2018) who use the bonds-in-utility assumption to analyze a unique secular stagnation scenario. Relative to this literature, our paper considers the two narratives of persistent ZLB and offers quantitative and analytical insights.

This paper is also related to the work by Mertens and Ravn (2014), Aruoba, Cuba-Borda and Schorfheide (2018), Bilbiie (2022), and Nakata and Schmidt (2021), who contrast expectations-driven and fundamental-driven liquidity traps using the standard Euler equation without discounting. Their setup can only accommodate a short-lived fundamentals-driven liquidity trap, while our modified Euler equation allows the possibility of secular stagnation as a competing hypothesis. Our paper is also complementary to Schmitt-Grohé and Uribe (2017), which analyzes the case of permanent expectations-driven liquidity traps. Coyle and Nakata (2019) characterizes optimal inflation target in the presence of expectations-liquidity traps. We build on these papers to show that policies that impose a lower bound on inflation preclude the expectations-driven traps. Benigno and Fornaro (2018)'s stagnation trap, which focuses on the role of pessimism about the economy's growth rate, is complementary to the inflation pessimism we study in this paper.

Our framework allows agents in the model to expect ZLB episodes of permanent duration under both hypotheses. This feature stands in contrast to models that use transitory declines in the natural interest rate to generate ZLB episodes where agents' expectations have to be consistent with recovery to the full-employment steady state in the medium run (Bianchi and

Melosi, 2017; Nakata, 2017; Nakata and Schmidt, 2019).

We use the principal-agent decision framework of [Del Negro et al. \(2016\)](#) to identify the relevant hypothesis in Japan, combining the predictive densities derived from observed time series. The Bayesian nature of our approach allows us to measure the uncertainty about the contrasting hypotheses with a structural model. Our prediction pool analysis is related to [Lansing \(2019\)](#) in which a model with endogenous regime switching generates data from a time-varying mixture of two models. In this paper, we construct a time-varying probability on the predictive densities of two alternative models. Our paper also relates to [Mertens and Williams \(2021\)](#) that uses the implications of changes in the natural interest rate on the distribution of interest rates and inflation in the options data from the U.S. financial markets, to discern between fundamentals- and expectations-driven liquidity. We find that the sign of inflation and output growth correlation can help distinguish between the hypotheses in a long-lasting liquidity trap.

Our analysis also builds on the important work by [Hirose \(2020\)](#) and [Iiboshi, Shintani and Ueda \(2022\)](#). [Hirose \(2020\)](#) estimates a model of deflationary-expectations-trap for Japan comparing equilibrium dynamics between the full-employment steady state before 1999 and the dynamics around the expectations-trap steady post-1999 in Japan. [Iiboshi et al. \(2022\)](#) uses non-linear methods to estimate a model in which the economy moves from the full-employment steady state into a temporary liquidity trap driven only by economic fundamentals. Relative to these estimated models for Japan, we tease out a testable implication to distinguish fundamental-driven from deflationary-expectations traps using the sign of an observed correlation of output growth and inflation. We show that this correlation allows us to separate the two hypotheses in Japanese data and is crucial for model fit.

A common theme of the papers that study expectation-driven liquidity traps is that policy implications may be the opposite of the ones derived from fundamentals-driven liquidity traps. As a result, it becomes central to assess what hypothesis is dominant in the data and develop policies that can always be stabilizing.<sup>3</sup> Our analysis is thus related to the design of robust policies such as fiscal policy rules that prevent the decline of real marginal costs

---

<sup>3</sup>One can develop expectations-traps equilibria with similar comparative statics as the fundamentals-driven liquidity traps, see [Eggertsson et al. \(2019, Figure 6 A\)](#). Our analysis does not focus on those, as they may not generate policy dilemmas.

(Schmidt, 2016), or fiscal stabilization policies that eliminate expectation-traps (Nakata and Schmidt, 2021). Similarly, research and development (R&D) subsidies advocated by Benigno and Fornaro (2018) that affect aggregate supply in an endogenous growth environment can eliminate expectations-driven liquidity traps.

**Layout.** Section 2 analytically teases testable predictions from both hypotheses. Section 3 presents our estimation results. In Section 4, we investigate the role of equilibrium selection. Section 5 extends our analysis to an estimated medium-scale DSGE environment. Section 6 concludes. All proofs and additional results are in the online appendix.

## 2. STYLIZED NEW KEYNESIAN MODEL

We begin with a stylized new Keynesian setup that entertains both the deflationary-expectations-driven liquidity trap and a permanent fundamentals-driven liquidity trap, labeled as *secular stagnation*. The model is a standard new Keynesian model augmented with a bonds-in-utility specification. The production side features monopolistically competitive firms facing nominal price rigidity. We use this setup to tease out a contrasting empirical implication of secular stagnation and deflationary expectation traps.

The central result is that secular stagnation generates a positive correlation between output growth and inflation. In contrast, the deflationary expectations trap can generate a negative correlation. The local indeterminacy of the expectations-trap equilibrium is key to obtaining a negative correlation.

### 2.1. Household

Time is discrete, and a representative agent maximizes the following lifetime utility:

$$\max_{\{C_t; h_t; B_{t+1}\}} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[ \log(C_t) - \frac{\omega}{1+\frac{1}{\eta}} h_t^{1+\frac{1}{\eta}} + \frac{\delta_t}{Z_t} \frac{B_{t+1}}{P_t} \right]$$

where  $C_t$  is consumption,  $h_t$  is hours supplied to work,  $\eta$  is the Frisch elasticity of labor supply,  $\omega$  is a constant set to normalize the steady state hours equal to one,  $B_{t+1}$  is the stock of one-period nominal risk-free government bonds,  $\delta_t \geq 0$  regulates the marginal utility from

holding the risk-free bonds supplied by the government, and  $Z_t$  is non-stationary level of total factor productivity (TFP) introduced in the utility function to get a stationary balanced growth path. Since our objective is to allow the possibility of a permanently negative natural rate, we adopt this modeling of bonds-in-utility following [Michaillat and Saez \(2021\)](#).<sup>4</sup>

The household earns nominal wage income  $W_t h_t$ , interest income on past bond holdings of risk-free government bonds  $B_t$  at gross nominal interest rate  $R_t$ , dividends  $D_t$  from firms' ownership, and receives transfers  $T_t$  from the government. The period-by-period budget constraint faced by the household is given by  $P_t C_t + \frac{B_{t+1}}{R_t} = W_t h_t + B_t + D_t + T_t$ . An interior solution to household optimization yields the consumption Euler equation

$$1 = \beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-1} \frac{R_t}{\Pi_{t+1}} \right] + \delta_t \left( \frac{C_t}{Z_t} \right). \quad (1)$$

where  $\Pi_t$  denotes the gross inflation rate.

## 2.2. Production

A perfectly competitive final-good-producing firm combines a continuum of intermediate goods indexed by  $j \in [0, 1]$  using the CES Dixit-Stiglitz technology:  $Y_t = \left( \int_0^1 Y_t(j)^{1-\nu_t} dj \right)^{\frac{1}{1-\nu_t}}$ , where  $1/\nu_t > 1$  is the time-varying elasticity of substitution across varieties, and  $\frac{1}{1-\nu_t}$  is the steady-state price markup. The shock  $\nu_t$  evolves according to:  $\log(\nu_t) = (1 - \rho_\nu) \log(\nu) + \rho_\nu \log(\nu_{t-1}) + \epsilon_{\nu,t}$ , where  $\epsilon_{\nu,t}$  are *iid*  $N(0, \sigma_\nu)$  innovations.

The price of the final good is given by  $P_t = \left( \int_0^1 P_t(j)^{\frac{\nu_t-1}{\nu_t}} dj \right)^{\frac{\nu_t}{\nu_t-1}}$  and profit maximization gives the demand for intermediate good  $j$  as a function of good  $j$ 's price relative to the final good price level:  $Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\frac{1}{1-\nu_t}} Y_t$ .

Intermediate good  $j$  is produced by a monopolist with a linear production technology:  $Y_t(j) = Z_t H_t(j)$ , where the growth rate of the aggregate TFP,  $G_{z,t}$ , follows:  $\log(G_{z,t}) \equiv \log Z_t - \log Z_{t-1} = (1 - \rho_z) \log(G_z) + \rho_z \log(G_{z,t-1}) + \epsilon_{z,t}$  and  $G_z$  denotes the steady state growth rate and  $\epsilon_{z,t}$  are *iid*  $N(0, \sigma_z)$  innovations.

Intermediate goods producers buy differentiated labor services  $H_t(j)$ , at a nominal price of

---

<sup>4</sup>As  $\delta \rightarrow 0$ , this equation nests the textbook Euler equation as a special case. We use the parameter  $\delta$  to target empirical estimates of the natural interest rate in Japan. The calibration will depend on the particular hypothesis, and we describe our strategy later in this section.



$W_t$ , and face quadratic adjustment costs when setting prices. These adjustment costs, expressed as a fraction of total output, are defined by the function  $\Phi_t \equiv \frac{p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \Pi \right)^2 Y_t$ . In a symmetric equilibrium, profit maximization yields the following price Phillips curve relation:<sup>5</sup>

$$(1 - \nu_t) - \omega h_t^{1-\nu_t} \frac{C_t}{Z_t} + \nu_t \Phi^\theta(\Pi_t) \Pi_t = \nu_t \beta E_t \left[ Q_{t+1} \Pi_{t+1} \Phi^\theta(\Pi_{t+1}) \frac{Y_{t+1}}{Y_t} \right] \quad (2)$$

### 2.3. Government and resource constraint

The desired policy rate is set according to the following rule  $\tilde{R}_t = \left[ r \Pi \left( \frac{P_t}{P_{t-1}} \right)^\pi \right]$ . Here,  $r$  is the steady-state real interest rate,  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate, and  $\Pi$  is the target inflation rate, which in equilibrium coincides with the steady-state inflation rate. The actual policy rate relevant to agents' decisions is subject to the zero lower bound constraint:

$$R_t = \max \left\{ 1, \tilde{R}_t \right\} \quad (3)$$

The government levies a lump-sum tax (subsidy) to finance any shortfalls in government revenues (or to rebate any surplus). The government's budget constraint is given by  $P_t G_t + B_t = T_t + \frac{B_{t+1}}{R_t}$ , where  $G_t = \left( 1 - \frac{1}{g_t} \right) Y_t$  captures autonomous sources of aggregate demand, including government expenditure. The shock  $g_t$  evolves according to:  $\log(g_t) = (1 - \rho_g) \log(g) + \rho_g \log(g_{t-1}) + \epsilon_{g,t}$ , where  $\epsilon_{g,t}$  are *iid*  $N(0, \sigma_g)$  innovations.

We assume that the price adjustment costs are rebated back to the household in a lump-sum fashion as part of the government transfers. The labor market clears:  $h_t = \int_0^1 H_t(j) dj$ , and the nominal bonds are in net zero supply  $B_t = 0$ . The market-clearing resource constraint is:

$$C_t + G_t = Y_t. \quad (4)$$

### 2.4. Steady-State Equilibrium and Calibration

The competitive equilibrium in the non-linear model is given by the sequence of four endogenous processes  $\{Y_t, C_t, R_t, \Pi_t\}$  that satisfy the conditions 1, 2, 3, and 4 for a given exogenous sequence of processes  $\{\delta_t, G_{z,t}, \nu_t, g_t\}_{t=0}^T$  and an initial level of aggregate TFP  $Z_0$ .

---

<sup>5</sup>We are substituting the aggregate labor market clearing condition:  $h_t = \int_0^1 H_t(j) dj$  in this derivation.

We define the steady-state natural interest rate as the real rate that prevails without nominal rigidities. The natural rate  $r$  is given by  $\frac{1}{\delta} - \beta$ . The natural rate (net) is negative if and only if  $\delta < 1 - \beta$ . It is non-negative otherwise. When the natural rate is positive, a unique positive interest rate steady state exists with output at potential and inflation at the central bank’s target.<sup>6</sup> We label this steady state as the *targeted* steady state.

Depending on the parameter values regulating the natural rate and the nominal rigidities, this framework admits two kinds of ZLB steady states. One is a liquidity trap steady state (à la [Schmitt-Grohé and Uribe, 2017](#)) that co-exists with the targeted steady state when the natural rate is positive and prices are not rigid enough. This steady state was discovered in the seminal work by [Benhabib et al. \(2001\)](#), and we label it as the *deflationary-expectations trap*. The main identifying feature of such a steady-state equilibria is that the dynamics in its neighborhood are locally indeterminate. It features inflation below the central bank’s target and output below potential. Pessimistic deflationary expectations can push the economy to this steady state without any change in fundamentals.

The second type of steady state with a binding zero lower bound emerges due to the presence of discounting in the consumption Euler equation and depends on the fundamentals driving the natural rate of interest. When the steady state natural rate is negative and nominal rigidities are severe enough, the economy can be permanently at zero nominal rates with below-target inflation and output below potential. We define the *secular stagnation steady state* as the steady state featuring a negative natural rate and zero nominal interest rate. The steady secular stagnation state exhibits locally determinate equilibrium dynamics in its neighborhood. This local determinacy property is the main difference between the secular stagnation narrative and the deflationary expectations-driven narrative that we seek to separate empirically.<sup>7</sup>

In our baseline model with forward looking Phillips curve, we numerically evaluate the steady state equilibria. In [Appendix E](#), we analytically prove the existence of the steady states of the model using a static Phillips curve. That analytical characterization shows that two parameters— $\delta$  (regulating the natural interest rate), and  $\phi_p$  (regulating the slope of the Phillips

---

<sup>6</sup>Potential output is defined as the level of output that would prevail under flexible prices.

<sup>7</sup>Note that multiple steady states at zero lower bound may coexist if the Phillips curve is sufficiently non-linear. Alternately, it may be possible to model the possibility of secular stagnation steady state coexisting with the full-employment steady state as in [Eggertsson et al. \(2019, Figure 6A\)](#) with a sufficiently high inflation target and a sufficiently non-linear Phillips curve.

curve)—determine the existence of the secular stagnation steady state and the deflationary expectations trap steady state.

**Table 1: Steady State Parameters**

<b>Externally Calibrated (Common Parameters)</b>					
$\beta$	$\eta$	$1/(1-\nu)$	$\Pi^*$	$g$	$G_z$
Discount factor	Inverse Frisch elasticity	Price markup	Inflation target	Autonomous spending	TFP growth rate
0.942	0.85	1.2	1.0025	1.81	0.25
<b>Endogenously Calibrated (Model-specific Parameters)</b>					
<b>Parameters</b>	<b>Secular stagnation</b>		<b>Expectations-trap</b>		
	$\delta$	$\phi$	$\delta$	$\phi$	
	Mg. utility bonds	Price adj. cost	Mg. utility bonds	Price adj. cost	
	0.1132	4825	0.1088	2524	
<b>Targets</b>	$r^*$	$\bar{\pi}$	$r^*$	$\bar{\pi}$	
	Natural Rate (a.r.)	Steady state deflation (a.r.)	Natural Rate (a.r.)	Steady state deflation (a.r.)	
	-1.1	-1.06	0	-1.06	

**Calibration.** The top part of Table 1 summarizes the steady-state parameters that are common across models. We fix the discount factor  $\beta$  to 0.942 consistent with structural estimates of Galí and Gertler (1999). While this estimate is lower than the standard calibrated value of 0.99 in the literature, a low  $\beta$  is needed for the model to generate a positive natural interest rate in the presence of a positive bond premium. In studies that have estimated the discount rate using field and laboratory experiments, the estimates for  $\beta$  are dispersed but point to high discount rates. Michailat and Saez (2021) choose an annual discount rate of 43% from the median value of these estimates in the experimental literature (Frederick, Loewenstein and O’Donoghue, 2002; Andersen, Harrison, Lau and Rutström, 2014).

We fix the Frisch labor supply elasticity at 0.85 following (Kuroda and Yamamoto, 2008). The elasticity of demand for intermediate goods  $1/\nu$ , is set to 6 to generate a steady-state markup of 20%. Japan did not officially adopt an inflation target until 2013:Q2, but the inflation rate averaged 1.1% in the two decades before entering the ZLB. Thus we assume the central

bank was pursuing an inflation target of 1% and use that target rate as the reference value for price adjustment ( $\Pi = 1.0025$ ).<sup>8</sup> We determine the values of steady-state TFP growth rate  $G_z$  such that the model matches the average output growth during the period 1998:Q1-2012:Q4. In the model, autonomous expenditure subsumes investment, net exports, and government spending. Consistent with this definition, we set  $g$  to match a consumption-output ratio of 55%.<sup>9</sup>

Two steady-state parameters,  $\delta$ , and  $\phi$ , are specific to each model. The bottom panel of Table 1 summarizes their calibration. These parameters match targets for Japan’s natural interest rate and average inflation. We adopt two targets for the natural rate depending on the regime. Under secular stagnation, we choose an annual rate of -1.1%. This choice is based on two studies by Fujiwara, Iwasaki, Muto, Nishizaki and Sudo (2016) and Iiboshi et al. (2022) that separately estimate a series for the natural interest rate in Japan based on Laubach and Williams (2003). They find that the quarterly estimate was often -0.5% since the late 1990s and -2% at the lowest level. In contrast, we calibrate the expectations-trap steady state to imply an annualized long-run real interest rate of 0%. Our calibrated values of the natural interest rate imply a unique value for  $\delta$  noted in Table 1. Using the Phillips curve equation 2, we calibrate  $\phi$  to match the average inflation rate of -1.06% for both steady states, which is the average inflation rate in Japan over our estimation sample period.<sup>10</sup> Our calibration results in a somewhat larger value of the price adjustment parameter,  $\phi$ , compared with econometric estimates of DSGE models for Japan (Iiboshi et al., 2022). Nonetheless, this value lies within the range of plausible estimates found in the literature—see Aruoba, Bocola and Schorfheide (2017). Finally, the implied output gap under both calibrations (not shown) is close to the estimates of 5% in Hausman and Wieland (2014).

## 2.5. Approximate Equilibrium

We approximate the equilibrium conditions (1–4) around secular stagnation and deflationary expectations trap steady states. We denote all liquidity-trap steady-state parameters by  $\bar{x}$  and

---

<sup>8</sup>Our results are robust to choosing a zero inflation target as well.

<sup>9</sup>As an alternative, it is straightforward to make  $g_t$  in the model track actual government spending in the data by defining consumption appropriately. Results are available upon request.

<sup>10</sup>Combined with zero nominal rates, this calibration targets the average real rate over the estimation sample.

denote  $\hat{x}_t$ , the log deviations of stationary variables relative to the steady state. Appendix A provides the derivation of the following log-linearized equations that summarize the dynamics of consumption, inflation, output, and the nominal interest rate:

$$\hat{c}_t = \bar{D}E_t(\hat{c}_{t+1} - \hat{R}_t + \hat{\pi}_{t+1} + \hat{G}_{z;t+1}) \quad (5)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \bar{\varphi} E_t (\hat{g}_{t+1} - \hat{g}_t) + \bar{\kappa} \left[ \left( \frac{1}{\eta} + 1 \right) \hat{y}_t - \hat{g}_t \right] + \bar{\lambda} \hat{\nu}_t \quad (6)$$

$$\hat{y}_t = \hat{c}_t + \hat{g}_t \quad (7)$$

$$\hat{R}_t = 0 \quad (8)$$

The coefficients entering equations 5 and 6 are functions of the following structural parameters:  $\bar{D} = \frac{1}{1 + G_z c}$ ;  $\bar{\kappa} = \frac{(1 - \phi) g c y^{1/\eta}}{2}$ ;  $\bar{\varphi} = \frac{\delta}{2}$ ; and  $\bar{\lambda} = \frac{1 - \phi}{2} \frac{(1 - \delta)}{2}$ , where  $\phi$  and  $\delta$  correspond to the cost of price adjustment and the marginal utility of bonds, these are the only structural parameters specific to each model. The shocks in the approximate equilibrium are (i) government expenditure,  $\hat{g}_t$ , (ii) the growth rate of productivity,  $\hat{G}_{z;t}$ , and (iii) price markups,  $\hat{\nu}_t$ , each following an AR(1) process.<sup>11</sup>

## 2.6. Properties of Approximate Equilibrium

We briefly discuss some analytical properties of the approximate equilibrium. The main takeaway is that secular stagnation generates a positive correlation between inflation and output growth while expectations trap can generate a negative correlation.

We begin with establishing the necessary and sufficient condition for determinacy in the approximate equilibrium.

**Proposition 1. (Local Determinacy).** Assume  $\beta < 1$ . The system 5 - 8 is locally determinate if and only if  $\frac{G_z c}{1 - \beta} > \frac{1 + \beta}{1 - \beta} \bar{\kappa}$ .

The secular stagnation steady state is defined to be a steady state at zero nominal interest

---

<sup>11</sup>A caveat of the deflationary expectations trap hypothesis is that it may not generate significant decline in real rate endogenously as the economy transitions from full employment regime to the liquidity trap regime. In our model, TFP growth rate shocks help match the observed real interest rate dynamics in Japan. Growth traps studied in Benigno and Fornaro (2018) present a complementary mechanism that can generate endogenous decline in real rate. They also note that the possibility of a self-fulfilling expectations trap is more likely when multiple sources of pessimism (growth, deflation) are allowed in the same model. We leave the quantitative analysis of this generalized hypothesis to future research.

rates which exhibits local determinacy. In Proposition 1, we provide conditions under which the approximate equilibrium exhibits local determinacy. Obtaining local determinacy requires a sufficiently flat Phillips curve (low  $\bar{\kappa}$ ), or high enough discounting (high  $\delta$ ).<sup>12</sup> In contrast, an expectations-driven liquidity trap has locally indeterminate dynamics, with low enough discounting or a sufficiently steep Phillips curve. Our steady-state calibrations for secular stagnation and expectations trap steady states to satisfy these restrictions.

Given the local determinacy, a unique solution under secular stagnation equilibrium can be derived. The solution is summarized in Proposition 2.

**Proposition 2. (Unique Solution under Secular Stagnation).** Let  $X = \{g, v, G_Z\}^\theta$  collect all fundamental state variables of the model, and let  $a$  and  $b$  be vectors of unknown coefficients. Assume the local determinacy condition in Proposition 1 is satisfied. The unique solution of the approximate equilibrium under secular stagnation is given by:

$$\hat{y}_t(X) = a_1 \hat{G}_{Z,t} + a_2 \hat{g}_t + a_3 \hat{v}_t; \quad \hat{\pi}_t(X) = b_1 \hat{G}_{Z,t} + b_2 \hat{g}_t + b_3 \hat{v}_t.$$

The coefficients  $(a_i, b_i)$  are reported in Appendix A.4.

Using this analytical solution for secular stagnation equilibrium, we establish in Proposition 3 that the correlation between inflation and output growth is positive.

**Proposition 3. (Positive Correlation under Secular Stagnation).** Consider the locally unique solution for the secular stagnation model shown in Proposition 2. The unconditional correlation between output growth and inflation is always positive.

The result follows from the fact that the steady-state aggregate demand relationship between inflation and output (derived from combining the Euler equation and the resource constraint) is upward-sloping under secular stagnation. Along with an upward-sloping Phillips curve, this steady-state equilibrium implies that price-markup shocks and TFP growth rate shocks only shift one schedule – either the Phillips curve or the Euler equation. These shifters unequivocally induce a positive correlation between inflation and output, given local determinacy. As long as the Phillips curve is sufficiently flat (low enough  $\bar{\kappa}$  relative to other structural parameters), the

---

<sup>12</sup>Definition 1 in [Michaillat and Saez \(2021\)](#) impose a similar restriction for obtaining a permanent fundamentals-driven ZLB episode.

government spending shock also induces a positive correlation between inflation and output, as well as inflation and output growth. The required restriction on the slope of the Phillips curve follows from the local determinacy restriction on parameters discussed in Proposition 1.

On the other hand, the deflationary expectations trap steady state features local indeterminacy. This implies that extraneous innovations, known as sunspot shocks ( $\zeta_t$ ), that are not part of the original description of agents' optimization problems, can determine the equilibrium outcomes (Lubik and Schorfheide, 2004; Canova and Gambetti, 2010). To characterize the multiplicity of equilibrium, we apply the methods in Bianchi and Nicolo (2021). This method allows one to index equilibria in locally indeterminate linear models by explicitly specifying a correlation structure between the i.i.d. sunspot shocks and the fundamental shocks.

Using their insight, we demonstrate that the expectations-trap model is capable of generating a negative correlation between inflation and output growth. Following Bilbiie (2019), we consider the following static Phillips curve to illustrate this result:

$$\hat{\pi}_t = \tilde{\kappa}\hat{y}_t + \bar{\lambda}\hat{\nu}_t \tag{9}$$

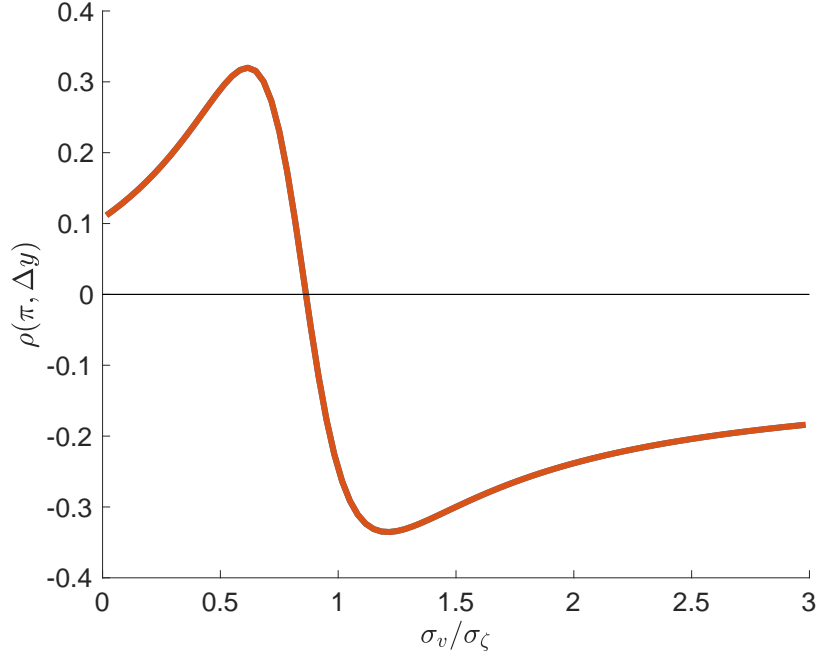
where we have shut down government spending shocks. For exposition reasons, we directly postulate this static Phillips curve and relegate its derivation from first principles to Appendix E. With this static Phillips curve, we can analytically show how a negative correlation between inflation and output growth emerges in several of the infinite solutions to the expectations-trap model.

It is instructive to work through the steps in this analytical case. Since the Phillips curve is assumed to be static, and we model the price-markup shocks as the only fundamental shocks, the system of equations (5), (7), (8), and (9) simplifies to a univariate system:

$$\hat{\pi}_t = \Lambda E_t \hat{\pi}_{t+1} + \bar{\lambda}\hat{\nu}_t$$

where  $\Lambda \equiv \bar{D}(1 + \tilde{\kappa})$ . Because we are analyzing the system around a locally indeterminate steady state, note that  $\Lambda > 1$ . We define the one-step ahead forecast error associated with the

**Figure 1:** *Inflation-Output Growth Correlation under Expectations-trap Hypothesis*



*Notes:* Theoretical correlations for stylized model under the following calibration:  $\bar{D} = 0.995, \bar{\lambda} = 0.25, \tilde{\kappa} = 0.1, \sigma_\zeta = 0.0030, \rho(\epsilon_\nu, \zeta) = 0.96$ .

expectational variable  $\hat{\pi}_t$ , as:<sup>13</sup>

$$\zeta_t \equiv \hat{\pi}_t - E_{t-1} \hat{\pi}_t \quad (10)$$

Since  $\Lambda > 1$ , we combine this equation with equation (10), to get the solutions of the following form:

$$\hat{\pi}_t = \Lambda^{-1} \hat{\pi}_{t-1} - \Lambda^{-1} \bar{\lambda} \hat{\nu}_{t-1} + \zeta_t$$

$$\hat{y}_t = \tilde{\kappa}^{-1} \hat{\pi}_t - \bar{\lambda} \tilde{\kappa}^{-1} \hat{\nu}_t$$

There are multiple solutions for the evolution of output, each indexed by the sunspot shock,  $\zeta_t$ . In Proposition 4, we show that this system can produce a negative correlation between inflation and output growth if the exogenous correlation between the sunspot and the fundamental shock is large enough.

<sup>13</sup>To be consistent with the idea of deflationary trap, we impose the sunspot on inflation forecast error. Appendix A.6 extends the proof to the case of sunspots on the output forecast error.



**Proposition 4.** (Negative Correlation under Deflationary-Expectations Trap). Consider the equilibrium of the expectations-trap model described by Equations (5), (7), (8), and (9) with only the i.i.d price-markup  $\hat{\nu}_t$  shocks as the only fundamental. Let  $\zeta_t \equiv \hat{\pi}_t - E_{t-1} \hat{\pi}_t$ , denote a mean zero sunspot shock with variance  $\sigma^2$ . Let  $\rho_{\zeta, \nu}$  denote the correlation between the sunspot and the markup shock. The correlation of output growth and inflation is negative if and only if  $1 > \rho_{\zeta, \nu} > \frac{\frac{2}{\zeta}}{(\frac{2}{\nu})}$ .

This result of a negative correlation between inflation and output growth with a static Phillips curve also extends to our baseline calibrated model with a forward-looking Phillips curve (Equation 6). To illustrate, we compute theoretical moments from the expectations-trap model assuming a correlation between sunspot and markup shocks equal to  $\rho(\epsilon_{\zeta}, \epsilon_{\nu}) = 0.96$ . In the subsequent section, we estimate this correlation with Japanese data. Figure 1 plots the correlation between output growth and inflation against the relative standard deviation of markup and sunspot shocks. In this example, for values of  $\frac{\sigma_{\nu}}{\sigma_{\zeta}} > 0.9$ , the expectations trap model generates a negative correlation between inflation and output growth.

In the rest of the paper, we investigate whether this particular correlation matters for model fit in the case of twenty years of near-zero interest rates in Japan.

### 3. MODEL EVALUATION DURING JAPAN'S LIQUIDITY TRAP

We now present a quantitative analysis based on the small-scale New Keynesian model of Section 2. We briefly discuss the estimation of remaining parameters and evaluate model fit using Bayesian methods.

#### 3.1. Data and estimation

Conditional on our calibration of steady state parameters, we estimate the vector of the following parameters  $\Theta = [\rho_g, \rho_z, \rho_{\zeta, \nu}, \sigma_g, \sigma_z, \sigma]'$ . For the expectations-trap model, in addition to the parameters listed in the vector  $\Theta$ , we also estimate the standard deviation of the sunspot shock,  $\sigma_{\zeta}$ , and the correlation between the structural and the sunspot shocks, denoted with  $\rho(x, \zeta)$ , for  $x = \{\epsilon_z, \epsilon_g, \epsilon_{\nu}\}$ . Because our model is linear we can construct the exact likelihood and use a standard Bayesian approach to estimate the parameters of the model. We obtain draws

from the posterior distribution by a single-block random walk Metropolis–Hastings (RWMH) algorithm (An and Schorfheide, 2007). Appendix C reports additional estimation details.

For parameter estimation, we use quarterly data on output growth, consumption growth, and GDP deflator-based inflation rate in Japan for the period 1998:Q1 – 2012:Q4.<sup>14</sup> We focus on this sample period for two reasons. First, from 1995 to 1998 the Bank of Japan (BOJ) held the monetary policy rate at 0.5%. We start our analysis in 1998 to parallel the assumption in our model that the economy starts at the ZLB and agents expect near-zero interest rates for a prolonged period. The BOJ lowered its policy rate to zero in the first quarter of 1999, and it remained between 0% and 0.5%. We consider the economy to be at the ZLB for the entire period. Second, in 2013, the BOJ introduced a new monetary policy program that included an explicit inflation target, asset and bond purchase programs as well as started considering negative nominal interest rates. None of these policies are explicitly modeled in our framework.

### 3.2. Estimation Results

Table 2 summarizes the estimated posterior distribution of parameters that fit the respective model to Japan’s output growth, consumption growth, and inflation data. The marginal prior distributions for the estimated parameters are tabulated in Appendix C.3. The posterior estimates for the common parameters are remarkably similar across model specifications. For the expectations-trap model, the standard deviation of the sunspot shock is statistically different from zero and with a magnitude similar to that of the shock to the technology growth rate. The estimated correlation between the fundamental and the sunspot shocks varies substantially. The data favors a robust positive correlation between the price markup and the sunspot shocks and a small correlation between the sunspot shock with the other two fundamental shocks.

The last row in Table 2 shows that the log-data density favors the expectations-trap hypothesis in terms of overall fit. To gauge the difference in fit, we construct Bayes factors of expectations-traps relative to secular stagnation,  $\mathcal{F} = p(Y^T|\mathcal{M}_b)/p(Y^T|\mathcal{M}_s)$ . As a test statistic, we compute  $2 \times \log(\mathcal{F})$  because it resembles the familiar likelihood-ratio test. In our estimation, we find that this test statistic is equal to 77, implying “very strong” evidence in

---

<sup>14</sup>Our findings are robust to using data from 1998:Q1-2019:Q4 in estimation. We use the longer sample for our assessment of the mechanism in section 4.

**Table 2:** Stylized DSGE Model: Posterior Estimates

Parameters	Description	$\mathcal{M}_b$ : Exp. trap		$\mathcal{M}_s$ : Sec. Stag.	
		Mean	[05 95]	Mean	[05 95]
$\rho_g$	Persistence gov. spending shock	0.91	[0.86 0.96]	0.88	[0.83 0.92]
$\rho_\nu$	Persistence markup shock	0.18	[0.08 0.28]	0.17	[0.08 0.25]
$\rho_z$	Persistence tech. growth shock	0.50	[0.27 0.74]	0.49	[0.25 0.72]
$100 \times \sigma_g$	Std dev. gov. spending shock	0.92	[0.81 1.04]	0.94	[0.82 1.06]
$100 \times \sigma_\nu$	Std dev. markup shock	0.35	[0.30 0.40]	0.38	[0.32 0.44]
$100 \times \sigma_z$	Std dev. tech. growth shock	0.36	[0.17 0.54]	0.64	[0.36 0.92]
$100 \times \sigma_\zeta$	Std dev. sunspot shock	0.36	[0.31 0.41]	-	-
$\rho(\epsilon_z, \zeta)$	Corr. sunspot and tech. growth shocks	-0.11	[-0.26 0.03]	-	-
$\rho(\epsilon_\nu, \zeta)$	Corr. sunspot and markup shocks	0.98	[0.96 1.00]	-	-
$\rho(\epsilon_g, \zeta)$	Corr. sunspot and gov. spending shocks	0.04	[-0.00 0.08]	-	-
$\log [p(Y^T)]$	Log-data density	-415.42		-453.90	

*Notes:* The estimation sample is 1998:Q1 - 2012:Q4. We use  $Y^T = [y_1, \dots, y_T]$  to denote all the available data in our sample. For each model, we report posterior means and 90% highest posterior density intervals in square brackets. All posterior statistics are based based on the last 50,000 draws from a RWMH algorithm, after discarding the first 50,000 draws.

favor of the expectations-trap hypothesis over secular stagnation according to standard criteria (Kass and Raftery, 1995).<sup>15</sup>

In our application, the sunspot shock and the correlation parameters necessary to select equilibria in the expectations-trap model come at a cost from the perspective of the log-data density as a penalty for additional parameters.<sup>16</sup> Nonetheless, one may still be concerned that the expectations-trap model always “edges over” secular stagnation because of the multiplicity of equilibria or the presence of “free” parameters. To allay this concern, we conduct an exercise where we simulate data from the secular stagnation model using the parameters in Table 2. Then, we re-estimate both models on simulated data and conduct a model comparison. We find that  $2 \times \log(\mathcal{F})$  is equal to  $-32$ , which indicates that when data comes from secular stagnation, our estimation procedure finds “very strong” evidence in its favor.

<sup>15</sup>According to Kass and Raftery (1995), values of  $2 \times \log(\mathcal{F})$  above 10 can be considered very strong evidence in favor of model 1. Values between 6 and 10 represent strong evidence, between 2 and 6 positive evidence, while values below 2 are “not worth more than a bare mention.”

<sup>16</sup>The log-data densities intrinsically penalize the likelihood function for the presence of additional parameters as in the Bayesian Information Criterion (Lubik and Schorfheide, 2004, Footnote 11).

### 3.3. Expectation traps or secular stagnation?

We now compare the relative importance of the two competing hypotheses in explaining the persistent liquidity trap episode in Japan over time. We use Bayesian prediction pools, as in [Geweke and Amisano \(2011\)](#) and [Del Negro et al. \(2016\)](#), that rely on predictive densities to construct recursive estimates of model weights. These time-varying model weights can be interpreted as a policymaker’s views on the most relevant model using the information available in real-time.

We consider a policymaker that has access to the sequence of one-period-ahead predictive densities  $p(y_t|y_{1:t-1}, \mathcal{M}_s)$  under secular stagnation and  $p(y_t|y_{1:t-1}, \mathcal{M}_b)$  under the expectations-trap hypothesis.<sup>17</sup> We are interested in constructing an estimate of the model weight,  $\lambda$ , that pools the information of each individual model:

$$p(y_t|\lambda, \mathcal{P}) = \lambda p(y_t|y_{1:t-1}, \mathcal{M}_b) + (1 - \lambda)p(y_t|y_{1:t-1}, \mathcal{M}_s), \quad 0 \leq \lambda \leq 1 \quad (11)$$

where  $p(y_t|\lambda, \mathcal{P})$  is the predictive density obtained by pooling the two competing models for a given weight  $\lambda$  and pool  $\mathcal{P} = \{\mathcal{M}_b, \mathcal{M}_s\}$ . The policymaker is Bayesian and has a prior density  $p(\lambda|\mathcal{P})$  of the weight assigned to each model in the pool. The posterior distribution of the model weights,  $p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \mathcal{P})$ , can be updated recursively conditional on the information available to the pool in the previous period  $\mathcal{I}_{t-1}^{\mathcal{P}}$ :

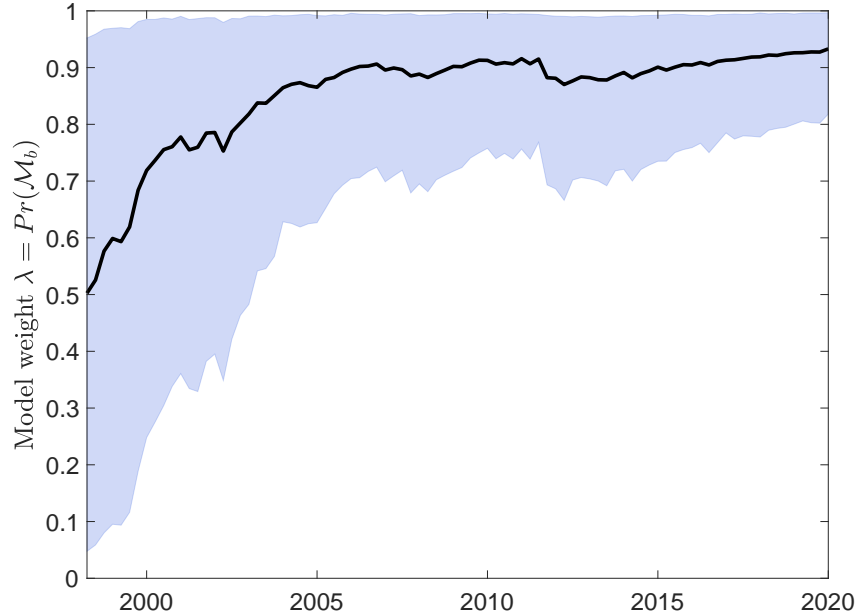
$$p(\lambda|\mathcal{I}_t^{\mathcal{P}}, \mathcal{P}) \propto p(y_t|\lambda, \mathcal{P})p(\lambda|\mathcal{I}_{t-1}^{\mathcal{P}}, \mathcal{P}) \quad (12)$$

We estimate the posterior distribution in Equation (12) recursively, starting in 1998:Q1. The estimated model weights are shown in [Figure 2](#) together with 90% posterior credible sets to capture model and parameter uncertainty. The Japanese data imply roughly similar weights on both models in the early part of the sample and through the early 2000s. Afterward, the data lean in favor of the specification  $\mathcal{M}_b$ , indicating a better fit of the expectations-trap hypothesis. Uncertainty about the model weight’s posterior distribution is substantial but decreases later in the sample as more information favoring the expectations-trap model accumulates. Starting in

---

<sup>17</sup>The predictive density is constructed sampling from the posterior distribution of the DSGE parameters of the baseline model of [Section 2](#) and averaging the predictive densities across draws.

**Figure 2:** *Model Weights: Expectations Traps vs Secular Stagnation*



*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2019:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

2015, the data put at least 90% weight on the expectations-trap hypothesis as the best-fitting explanation.

## 4. INSPECTING THE MECHANISM

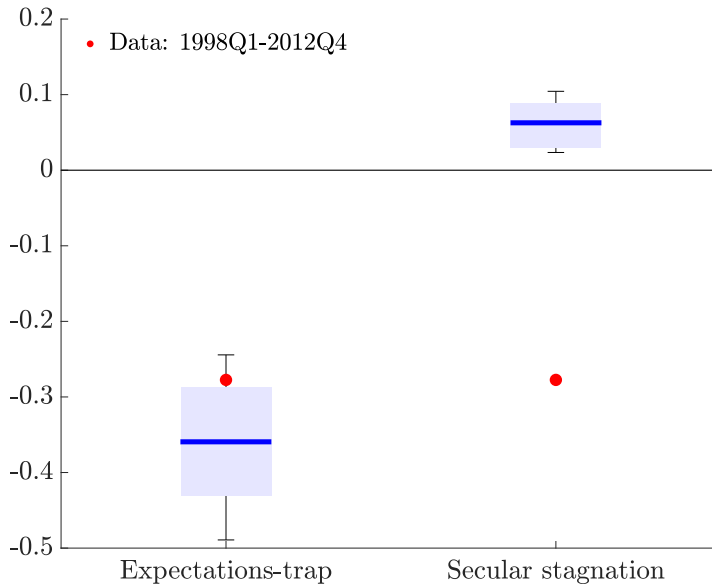
### 4.1. Correlation between Inflation and Output Growth

We first discuss the data moments that favor the expectations-trap hypothesis in our estimation. We find that the superior model fit of the expectations trap is related to its ability to generate a negative unconditional correlation between inflation and output growth, as observed in the data.

The two columns in Figure 3 show the range of theoretical correlations between inflation and output growth implied by the posterior parameter distribution of the expectation traps and secular stagnation model respectively. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively. The red dots in the figure represent the same moments in the Japanese data used in estimation.

The expectations-trap model generates a negative unconditional correlation between inflation

**Figure 3:** Moments: models vs data



*Notes:* Dots correspond to sample moments in Japan’s data. Solid horizontal lines indicate medians of theoretical moments of the posterior distributions for parameter estimates. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively.

and output growth consistent with the data. In contrast, as shown analytically in Proposition 3, the secular stagnation model can only generate a positive inflation output growth correlation.<sup>18</sup>

We next explore the source of this negative correlation in the expectations trap model.

## 4.2. Indeterminacy and sunspot shocks

The presence of equilibrium indeterminacy relaxes the tight co-movement between inflation and output that afflicts the secular stagnation model, and inflation in the expectations-trap model can arbitrarily jump in response to fundamental shocks. In section 2, we introduced i.i.d. sunspot shocks as exogenous expectational errors on inflation to select among the multiple equilibria. Using the technique of Bianchi and Nicolò (2021), we allowed sunspot shocks to be correlated with fundamental shocks in the model to index various equilibria. We now investigate which correlations with structural shocks are essential for our results. And how do our results change if we discipline the equilibrium selection with data on inflation expectations?

<sup>18</sup>Datta, Johannsen, Kwon and Vigfusson (2021) document a positive correlation between oil and equity prices in the U.S. post-2008. This measure is a proxy of the correlation between inflation and output growth in our model. We leave a formal quantitative assessment for the U.S. in our framework for future work.

### 4.2.1 Which Equilibrium?

The correlation between sunspots and structural shocks is crucial because it characterizes all admissible solutions under indeterminacy (Bianchi and Nicolo, 2021) while disciplining equilibrium selection using data.<sup>19</sup>

To understand which of the multiple equilibrium paths plays a role in discriminating between expectation traps and secular stagnation, we re-estimate the prediction pool under four restrictions on the correlation between the sunspot and fundamental shocks. Figure 4 displays the estimated time-varying model weights. Panel (a) sets the correlation between the sunspot and productivity shocks to zero. Panel (b) sets the correlation between the sunspot shock and the government expenditure shock to zero. Panel (c) sets the correlation of the sunspot shock and markup shock to zero. Lastly, panel (d) sets all the correlations to zero.

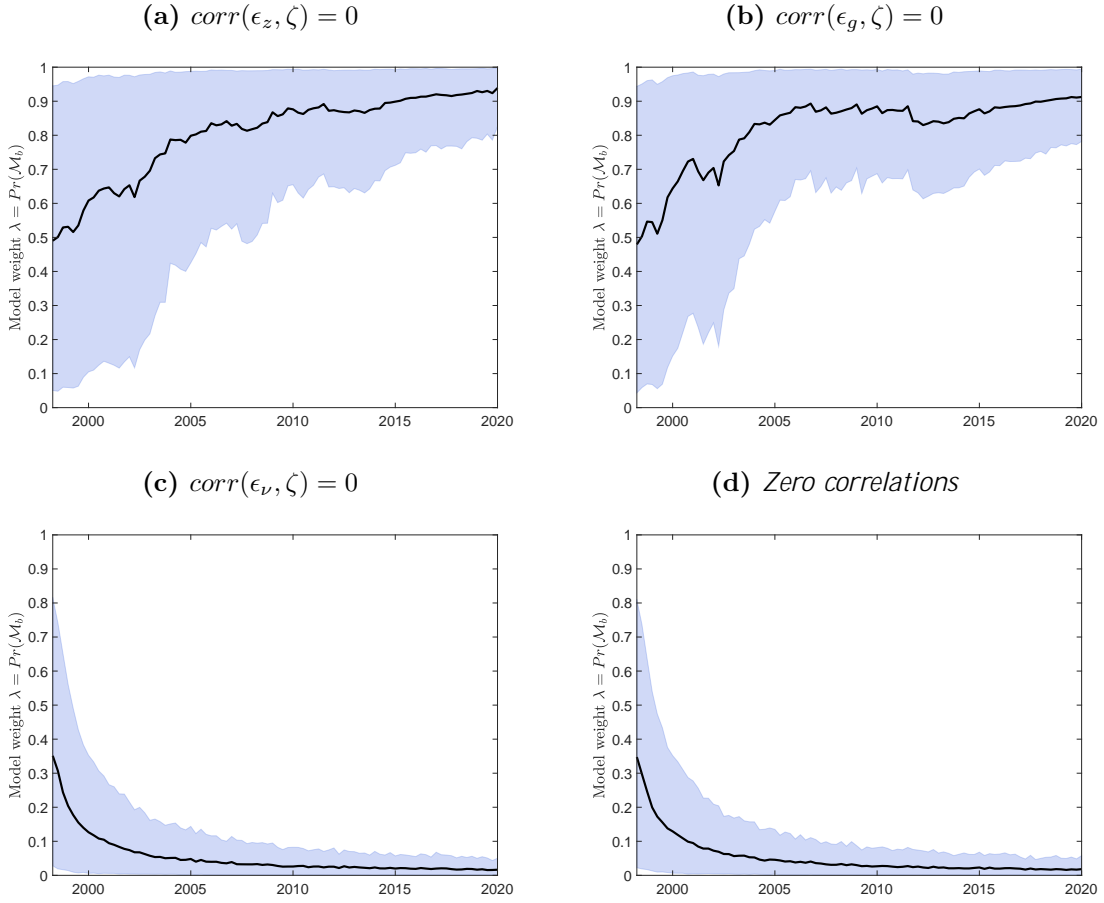
When price-markup and sunspot shocks are uncorrelated, as in panels (c) and (d), secular stagnation explains the data better. Conversely, when the correlation between sunspots with productivity or government spending shocks is zero while letting the data choose the correlation between sunspot and markup shocks, we obtain results similar to our baseline specification, i.e., expectation traps as the more likely explanation for the observed data. Combined with our analytical results presented in Section 2, these equilibrium selection results imply that the correlation between price markup and sunspot shocks is crucial for the expectations trap hypothesis because it allows the model to generate a negative correlation between inflation and output growth. This correlation implies that markup shocks are contractionary at the zero lower bound—see impulse responses in Appendix C.5. This finding echoes the evidence presented in Wieland (2019), which shows that the oil supply shocks in Japan, which are equivalent to price markup shocks in our model, generate a negative correlation between inflation and output at the ZLB.<sup>20</sup>

---

<sup>19</sup>While we discipline equilibrium selection using the empirical model fit, a caveat of expectation traps is that we do not explicitly model how this expectations formation occurs, and how agents coordinate on one of the many multiple equilibria.

<sup>20</sup>Relatedly, Cohen-Setton, Hausman and Wieland (2017) provide partial equilibrium evidence from a supply-side policy restricting hours worked in France. They find that a policy-mandated reduction in weekly hours worked adversely affected industrial production in sectors exposed to the law.

**Figure 4: Model Weights: Role of Sunspots**



*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

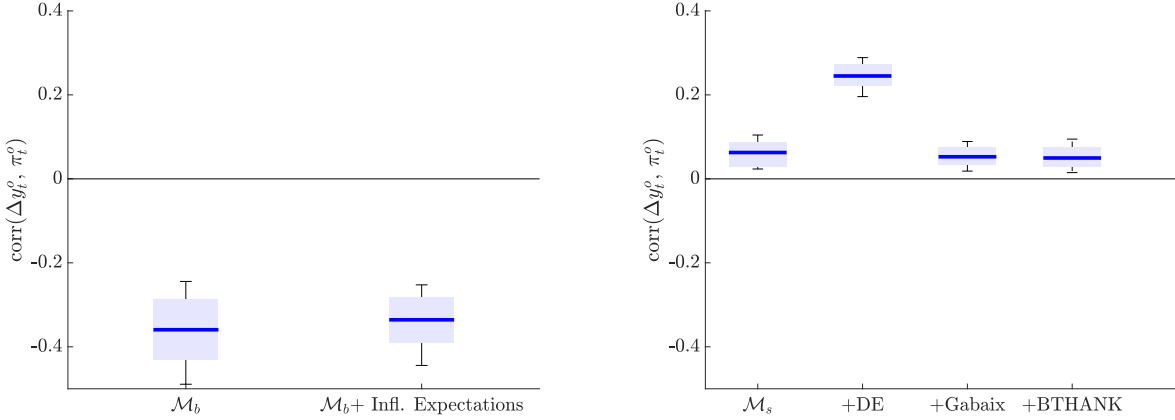
#### 4.2.2 Inflation Expectations Data and Indeterminacy

We now investigate if data on inflation expectations as a source of additional information helps discipline equilibrium selection in the expectations-trap model. We collect Japan's 6- to 10-year inflation expectations data from Consensus Economics. Inflation expectations data is semi-annual and we use linear interpolation to obtain quarterly figures. We relate quarterly deviations from the sample mean in the data to the implied steady-state deviations of 6- to 10-year inflation expectations in the model. Formally we construct:  $\hat{\pi}_t^e = \frac{1}{20} \mathbb{E} \left[ \sum_{h=21}^{40} \hat{\pi}_{t+h} \right]$  and augment the expectations-trap model with the following measurement equation  $\pi_t^{e:o} = 400 \times \hat{\pi}_t^e$  and re-estimate the posterior distribution of parameters associated with the expectations-trap model. Detailed estimation results are presented in Appendix C.6.



**Figure 5: Robustness: in ation-output correlation**

(a) *Expectations-trap with in ation expectations data* (b) *Secular stagnation without rational expectations*



*Notes:* Solid horizontal lines indicate medians of theoretical moments of the posterior distributions for parameter estimates. Boxes and whiskers indicate 90% and 99% credible sets of the posterior distributions, respectively.

The left panel in Figure 5 compares the posterior distribution of the theoretical correlation between inflation and output growth in our benchmark estimation and in the expectations-trap model re-estimated using data on inflation expectations. When using the data on inflation expectations, the model matches this key empirical correlation between inflation and output growth. Consistent with tighter posterior intervals on the estimated correlation parameters, reported in Table 5 in Appendix C.6, we recover a somewhat tighter posterior range for correlation between inflation and output growth relative to the first column where we do not use data on inflation expectations in the estimation.

Since the expectations trap model is estimated using additional data, we cannot directly compare the log data density with the baseline secular stagnation model. Nonetheless we construct the Bayesian prediction pools to study which model explains the observed data series better. In Appendix C.7, we report these results. We find that the expectations trap model, even when estimated with additional data restrictions, continues to be superior to the secular stagnation model in explaining the three data series used in the baseline estimation.

### 4.3. Secular stagnation without rational expectations

We investigate the ability of departures from the rational expectations assumption in their ability to enhance secular stagnation’s empirical fit relative to the expectations trap hypothesis.

We consider three dominant alternatives: (i) diagnostic agents à la [Bordalo, Gennaioli and Shleifer \(2018\)](#), (ii) “behavioral” agents à la [Gabaix \(2020\)](#), and (iii) behavioral heterogeneous agents à la [Bilbiie \(2021\)](#) and [Pfäuti and Seyrich \(2022\)](#). The stagnation steady state is unchanged in all three extensions. We provide a detailed derivation of the log-linearized equilibrium conditions of these model extensions in [Appendix B](#), while additional estimation results are available in [Appendix C.6](#)

We find neither of these departures from rational expectations allows secular stagnation equilibrium to match the negative correlation between inflation and output growth. Our first extension considers diagnostic expectations, in which agents extrapolate current innovations thus distorting beliefs about future states.<sup>21</sup> Relative to the baseline, there is a new parameter  $\theta \geq 0$  that regulates the departure from rational expectations. A value of  $\theta = 0$  simplifies the model to the rational expectations benchmark. We set  $\theta = 1$ , following [Bordalo et al. \(2018\)](#). The second column in [Figure 5b](#) shows that the estimated secular stagnation model with diagnostic expectations also exhibits a positive correlation between output growth and inflation. The marginal data density of this model is 20 log points lower than that of the rational expectations counterpart.

In contrast to extrapolation, [Gabaix \(2020\)](#) proposes a model of “cognitive discounting”, whereby the representative agent is partly myopic about innovations in the distant future.<sup>22</sup> In this model we introduce a new parameter,  $\bar{m} \in [0, 1]$ , that regulates the behavioral discounting. Following [Gabaix \(2020\)](#), we set  $\bar{m} = 0.85$ , and re-estimate the baseline secular stagnation model with this configuration. The case of  $\bar{m} = 1$  reduces the model to the rational expectations benchmark. The third column in [Figure 5b](#) shows that the estimated secular stagnation model with cognitive discounting exhibits a lower correlation between output growth and inflation than the baseline case in the first column. The marginal data density of this model is 2 log points higher than the benchmark secular stagnation model with rational expectations. Still, the model fit remains substantially lower relative to the expectations-trap model.

Finally, we consider [Bilbiie \(2021\)](#)’s tractable heterogeneous agents New Keynesian (THANK) model combined with [Gabaix \(2020\)](#) behavioral friction as formally studied by [Pfäuti and](#)

---

<sup>21</sup>We follow [Bianchi, Ilut and Saijo \(2023\)](#) and [L’Huillier, Singh and Yoo \(2023\)](#) in their approach to solving diagnostic expectations in linear general equilibrium settings.

<sup>22</sup>[Angeletos, Huo and Sastry \(2020, Sec. 6.4\)](#) show that leading departures from rational expectations such as level-k thinking, dogmatic higher-order beliefs, and cognitive discounting imply under-extrapolation.

Seyrich (2022). The resulting behavioral-THANK (B-THANK) model analytically captures key ingredients of rich heterogeneity often considered in quantitative models while allowing for local determinacy through cognitive discounting. To keep the steady state unchanged, we consider the baseline version in Bilbiie (2021) featuring a zero-inequality steady state which isolates the role of cyclical inequality. The steady state of the model is thus the same as before. The new parameters  $\chi = 1.48$  (a composite measure of cyclical income inequality),<sup>23</sup>  $\lambda = 0.37$  (an unconditional measure of “hand-to-mouth” agents), and  $1 - s = 0.04$  (transition probability to move from being a saver to a “hand-to-mouth” agent) are set following Bilbiie (2021)’s baseline calibration. The final column in Figure 5b shows that the estimated secular stagnation model with cognitive discounting and heterogeneous agents also exhibits a somewhat lower correlation between output growth and inflation than the baseline case depicted in the first column. However, the marginal data density of this model is 7 log points lower than the representative rational agent counterpart.

In sum, while the cognitive discounting friction offers a better model fit at the margin for the secular stagnation model, neither of the three departures allows the secular stagnation model to generate the negative correlation between inflation and output growth observed in the data. Our results suggest that the fit of the secular stagnation hypothesis is impaired because the model cannot generate the empirical correlation of inflation and output growth even with some of the departures from rational expectations.<sup>24</sup>

## 5. A MEDIUM-SCALE DSGE MODEL

We assess the robustness of our findings using a medium-scale model similar to Smets and Wouters (2007), which extends our benchmark New Keynesian model along several dimensions. We use this model to explore two dimensions of our results. First, the medium-scale model features additional cross-equation restrictions and data relative to the model in Section 2. Do additional complexity and more data diminish the role of the negative correlation between output growth and inflation as the key moment driving model fit? Second, we use the quantitative

---

<sup>23</sup> $\chi > 1$  is the empirically relevant case of counter-cyclical income inequality in this framework.

<sup>24</sup>It is possible to relax model misspecification by allowing the correlation of fundamental shocks in the secular stagnation model, thus generating a negative inflation-output correlation. We do not see a clear economic interpretation to pursue such an approach.

model to ask if the correlation of markup shocks with sunspot shocks is still one of the main drivers of the model fit for the expectations trap. Relaxing misspecification through a more elaborate model structure or additional data does not overturn the main insights we derived with the baseline model.

The medium-scale extension of the baseline model is relatively standard. We briefly describe the main ingredients and defer a detailed derivation to Appendix F. There are three important differences relative to the model in Section 2. First, we allow for internal habits in consumption. Second, we introduce nominal wage stickiness as in [Erceg, Henderson and Levin \(2000\)](#). Third, we introduce capital into production, with costly capital utilization and investment adjustment costs as in [Christiano, Eichenbaum and Evans \(2005\)](#). Along each of these modifications, we introduce three new structural shocks: time-varying wage markups ( $\frac{1}{1+w,t}$ ), a risk-premium shock ( $\eta_t$ ), and a shock to the marginal efficiency of investment ( $\mu_t$ ) as in [Justiniano, Primiceri and Tambalotti \(2010\)](#).

## 5.1. Data and Calibration

We follow [Hirose \(2020\)](#) to map the medium-scale model to Japanese data. Data for estimation includes time series on output growth, personal consumption expenditure growth, investment growth, GDP deflator-based inflation, wage growth, and hours worked. As in the analysis of Section 3, we estimate the relevant parameters in the medium-scale model using data from 1998:Q1 - 2012:Q4, while we use the sample 1998:Q1 - 2019:Q4 to conduct the prediction pool analysis and assess the likelihood of expectation-traps vis-a-vis secular stagnation. Detailed data sources are available in Appendix C.

The parameters that govern the capital share in production, depreciation rate of capital, habit persistence in consumption, price and wage indexation, capital utilization elasticity, and investment adjustment costs are sourced from [Hirose \(2020\)](#). To be consistent with the stylized model of Section 3, we have set the discount factor  $\beta = 0.942$ , and calibrated the steady-state level of price and wage markups to 20 percent. Likewise, we have set the growth rate of productivity and the share of government expenditure to GDP to their observed values during the estimation sample from 1998:Q1 to 2012:Q4. For a detailed description of model calibration, please refer to Appendix F.

We separately estimate the parameters governing each model’s structural shocks and equilibrium selection. The expectations-trap and secular stagnation models have six structural shocks in common, labeled as follows: technology growth shocks ( $G_{z,t}$ ); domestic absorption shocks ( $g_t$ ), which include government spending and foreign demand; shocks to the marginal efficiency of investment ( $\mu_t$ ); shocks to price markup ( $\nu_{p,t}$ ) and wage markup ( $\nu_{w,t}$ ); and shocks to households risk premia ( $\eta_t$ ) that shift the marginal utility of bonds. All the shocks follow a first-order auto-regressive process. The expectations-trap model has seven additional parameters: a non-fundamental or sunspot shock ( $\zeta_t$ ) and six parameters for the correlation of the sunspot shock with the structural shocks that select the equilibrium path under indeterminacy.

## 5.2. Estimation Results

Table 3 reports the estimated parameters for both models. We report posterior mean estimates and 90% credible sets. In the bottom row, we report the marginal likelihoods of both models. Additional estimation details, such as the prior distribution and the configuration of the posterior sampler, are available in Appendix C.

The estimated posterior mean for the common parameters across the two models is similar. In terms of overall fit, although the medium-scale model has additional propagation and shocks, we still find “strong” evidence favoring the expectations-trap model over secular stagnation. The Bayes factor is 17. The experiments in the remainder of this section suggest that this difference in fit is also related to the central moment of our analysis, the negative correlation between inflation and output growth when interest rates are at the ZLB.

Turning to the estimated parameters specific to the expectations-trap model, we make two observations. First, the sunspot shock’s standard deviation is similar to our estimates in Table 2. This result suggests that sunspot shocks remain a statistically important source of fluctuations despite the additional structural shocks in the model. Second, we find that the correlation of the sunspot shock with price and wage markup shocks is positive and tightly estimated. In particular, the mean estimate of the correlation between sunspot and price markup shock is remarkably close to our estimates in the stylized model of Section 3. Both results suggest that parameters governing the dynamics of the expectations trap model are robustly estimated using additional data and a more complex model.

**Table 3:** *Medium Scale Model: Posterior Estimates*

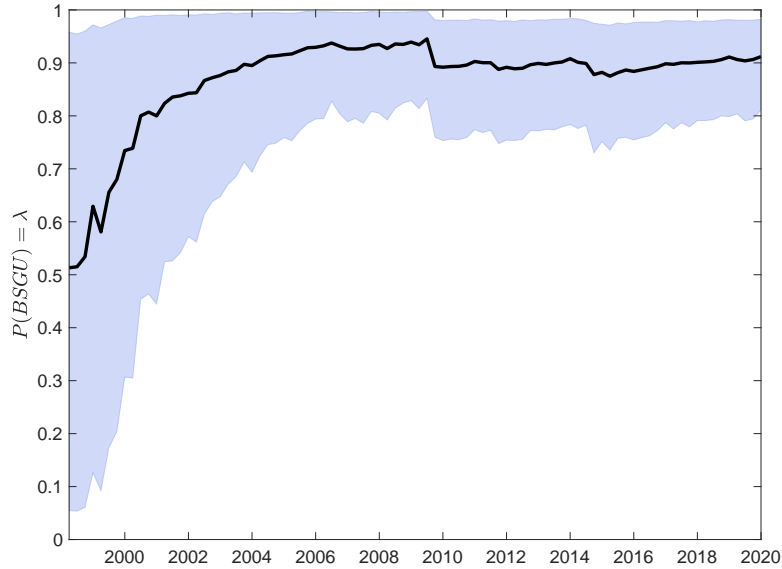
Parameters	Description	$\mathcal{M}_b$ : Exp. trap		$\mathcal{M}_s$ : Sec. Stag.	
		Mean	[05 95]	Mean	[05 95]
$\rho_w$	Persistence wage markup shock	0.20	[0.19 0.21]	0.17	[0.06 0.27]
$\rho_p$	Persistence price markup shock	0.20	[0.19 0.21]	0.09	[0.03 0.15]
$\rho_g$	Persistence gov. spending shock	0.99	[0.98 1.00]	0.89	[0.85 0.93]
$\rho_\mu$	Persistence MEI shock	0.42	[0.41 0.43]	0.90	[0.87 0.94]
$\rho_\eta$	Persistence risk premium shock	0.35	[0.34 0.37]	0.87	[0.82 0.93]
$\rho_z$	Persistence tech. growth shock	0.06	[0.05 0.06]	0.18	[0.10 0.26]
$100 \times \sigma_w$	Std dev. wage markup shock	0.55	[0.49 0.60]	0.49	[0.41 0.56]
$100 \times \sigma_p$	Std dev. price markup shock	0.49	[0.47 0.52]	0.48	[0.42 0.54]
$100 \times \sigma_g$	Std dev. gov. spending shock	0.72	[0.65 0.77]	0.77	[0.67 0.86]
$100 \times \sigma_\mu$	Std dev. MEI shock	14.17	[12.01 16.16]	7.94	[6.87 8.91]
$100 \times \sigma_z$	Std dev. tech. growth shock	1.49	[1.32 1.66]	1.43	[1.24 1.62]
$100 \times \sigma_\eta$	Std dev. risk premium shock	0.57	[0.14 1.16]	4.01	[2.08 5.87]
$100 \times \sigma_\zeta$	Std dev. sun. shock	0.49	[0.47 0.51]	-	-
$\rho(\epsilon_z, \zeta)$	Corr. sun. and tech. growth shocks	-0.04	[-0.06 -0.02]	-	-
$\rho(\epsilon_g, \zeta)$	Corr. sun. and gov. spending shocks	-0.08	[-0.09 -0.06]	-	-
$\rho(\epsilon_\mu, \zeta)$	Corr. sun. and MEI shocks	-0.09	[-0.11 -0.07]	-	-
$\rho(\epsilon_p, \zeta)$	Corr. sun. and price markup shocks	0.95	[0.93 0.96]	-	-
$\rho(\epsilon_w, \zeta)$	Corr. sun. and wage markup shocks	0.29	[0.25 0.32]	-	-
$\rho(\epsilon_\eta, \zeta)$	Corr. sun. and risk premium shocks	0.07	[0.05 0.09]	-	-
$\log [p(Y^T)]$		-944.67		-953.07	

*Notes:* The estimation sample is 1998:Q1 - 2012:Q4. We use  $Y^T = [y_1, \dots, y_T]$  to denote all the available data in our sample. We report posterior means and 90% the highest posterior density intervals for each model in square brackets. All posterior statistics are based on the last 50,000 draws from an RWMH algorithm after discarding the first 50,000 draws.

### 5.3. Expectations-trap vs secular stagnation

Next, we construct recursive weights using the predictive densities from the medium-scale specification of both models. Figure 2 shows the posterior mean estimates of the model weights,  $\lambda_t$ , and the associated 90 percent credible sets. The estimated value of  $\lambda_t$  reflects the probability that the six data series observed in period- $t$  come from the expectations-trap hypothesis. Our results confirm that both models provided an equally plausible explanation of the data in the initial part of the sample. With additional information accumulating over time, the expectations-trap model emerges as a more likely explanation, with our estimates of  $\lambda_t$  rising to about 90 percent by 2010 and remaining at that level. Although the estimated mean probability of the expectation-trap model kept rising over time, uncertainty about the source of stagnation remained elevated for several years.

**Figure 6:** *Expectations Traps vs Secular Stagnation in Medium Scale Model*



*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2019:Q4. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

#### 5.4. Discussion

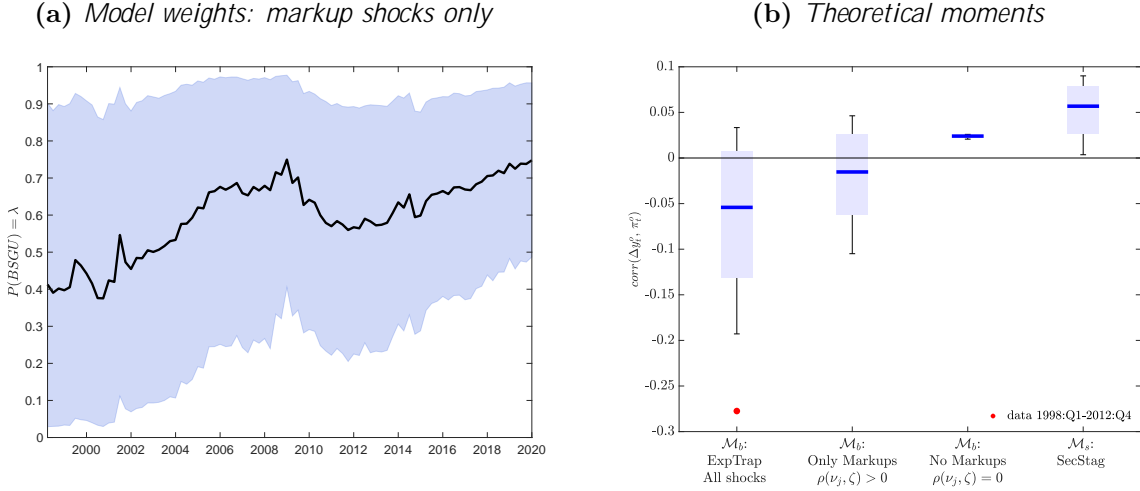
Using a small-scale model, we argued that the correlation between inflation and output growth is central to tilting the prediction pool weights in favor of the expectations trap. Using the estimated medium-scale model, we now revisit the importance of the correlation between inflation and output growth.

In Figure 7, we re-estimate the model weights,  $\lambda_t$  while restricting the correlation structure between the sunspot and the structural shocks. In all experiments, we draw the estimated parameters from the posterior distributions presented in Table 3 but set correlations between the sunspot and particular structural shocks to zero when computing predictive densities.

The left panel in Figure 7 shows the estimated model weights when the sunspot shock is allowed to have a non-zero correlation with the price and the wage markup shocks.<sup>25</sup> The mean estimates of  $\lambda_t$  indicate that the expectations-trap model remains the preferred model. Although the weight of the expectations-trap model increases steadily over our sample, the

<sup>25</sup>In Appendix C.7 we present the reverse exercise in which we restrict the correlation of the sunspot shock with the markup shocks to be zero and allow the correlation of sunspot shocks with other structural shocks to be unrestricted. We find that the recursive model weights favor the secular stagnation model, consistent with our results from the estimated small-scale model. The correlation between sunspot and markup shocks is the key driver for model fit in the expectations-trap model.

**Figure 7:** *Inspecting the Mechanism in the Medium Scale Model*



*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2020:Q1. The shaded areas correspond to the 90 percent credible set of the posterior distribution.

wider credible sets show more uncertainty about the dominant model.

We find that the correlation between inflation and output growth remains the key moment in the tilt of model weights toward the expectations-trap model. The right panel in Figure 7 shows the theoretical correlation between inflation and output growth for three versions of the expectations-trap model. Specifically, we plot the baseline estimates (first column), a restricted model in which sunspot shocks are correlated only with markup shocks (second column), and a restricted model in which the correlation of sunspot shocks and markup shocks is zero (third column). For comparison, we also report the theoretical moments for the secular stagnation model (fourth column).

In the unrestricted expectations-trap model, shown in the first column, the median correlation between inflation and output growth is negative and the posterior intervals cover a large range of values. In contrast, the secular stagnation model has a very tight range of outcomes in the positive territory for this central correlation. In the restricted expectations trap model with only markup shocks driving the equilibrium selection, shown in the second column, the range of correlations between inflation and output growth narrows but the posterior mean estimate remains negative. In the third column, the restricted model with zero correlation between markup and sunspot shocks, the mean correlation between inflation and output growth is tightly distributed around a positive value. These results suggest that equilibrium selection through markup shocks is the key source of the correlation between inflation and output growth.



Overall, our quantitative findings from the medium-scale model align with the analytical and quantitative results from the small-scale model despite the added model complexity.

## 6. CONCLUSION

In this paper, we formally test two hypotheses of stagnation: deflationary expectations traps and secular stagnation. We entertain both models within a unified framework using a modified Euler equation with discounting. Because both theories differ in their local determinacy properties, we show analytically that the correlation of output growth and inflation can distinguish the two hypotheses in the data.

We leverage the Japanese experience at the ZLB to empirically assess our theory using a quantitative New Keynesian model that embeds both hypotheses. We offer a summary measure of the likelihood of each hypothesis using Bayesian prediction pools. We find evidence that Japan’s experience is consistent with the expectations trap model, but making such an assessment is subject to substantial uncertainty in the early part of the estimation sample. Consistent with our theory, the negative correlation between output growth and inflation is the key empirical moment that ultimately shifts the balance in favor of the expectations-trap hypothesis in Japan. Our quantitative findings extend to models that deviate from rational expectations, models that use inflation-expectations data, and more elaborate medium-scale models of the Japanese economy.

## REFERENCES

- Andersen, Steffen, Glenn W Harrison, Morten I Lau, and E Elisabet Rutström.** 2014. “Discounting behavior: A reconsideration.” *European Economic Review*, 71: 15–33.
- Angeletos, George-Marios, Zhen Huo, and Karthik A. Sastry.** 2020. “Imperfect macroeconomic expectations: Evidence and theory.” In *NBER Macroeconomics Annual 2020*, Vol. 35.
- An, Sungbae, and Frank Schorfheide.** 2007. “Bayesian analysis of DSGE models.” *Econometric reviews*, 26(2-4): 113–172.
- Aruoba, Boragan, Luigi Bocola, and Frank Schorfheide.** 2017. “Assessing DSGE model nonlinearities.” *Journal of Economic Dynamics and Control*, 83: 34–54.

- Aruoba, S Borağan, Pablo Cuba-Borda, and Frank Schorfheide.** 2018. “Macroeconomic dynamics near the ZLB: A tale of two countries.” *The Review of Economic Studies*, 85(1): 87–118.
- Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe.** 2001. “The Perils of Taylor Rules.” *Journal of Economic Theory*, 96(1): 40–69.
- Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe.** 2002. “Avoiding liquidity traps.” *Journal of Political Economy*, 110(3): 535–563.
- Benigno, Gianluca, and Luca Fornaro.** 2018. “Stagnation Traps.” *The Review of Economic Studies*, 85(3): 1425–1470.
- Bhattarai, Saroj, Gauti B Eggertsson, and Bulat Gafarov.** 2022. “Time consistency and duration of government debt: A model of quantitative easing.” *The Review of Economic Studies*, rdac063.
- Bianchi, Francesco, and Giovanni Nicolo.** 2021. “A Generalized Approach to Indeterminacy in Linear Rational Expectations Models.” *Quantitative Economics* (forthcoming).
- Bianchi, Francesco, and Leonardo Melosi.** 2017. “Escaping the great recession.” *American Economic Review*, 107(4): 1030–58.
- Bianchi, Francesco, Cosmin Ilut, and Hikaru Saijo.** 2023. “Diagnostic Business Cycles.” *The Review of Economic Studies*, rdad024.
- Bilbiie, Florin O.** 2019. “Optimal Forward Guidance.” *American Economic Journal: Macroeconomics*, 11(4): 310–45.
- Bilbiie, Florin Ovidiu.** 2021. “Monetary Policy and Heterogeneity: An Analytical Framework.” CEPR Discussion Papers 12601.
- Bilbiie, Florin Ovidiu.** 2022. “Neo-Fisherian Policies and Liquidity Traps.” *American Economic Journal: Macroeconomics*, 14(4): 378–403.
- Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer.** 2018. “Diagnostic expectations and credit cycles.” *The Journal of Finance*, 73(1): 199–227.
- Canova, Fabio, and Luca Gambetti.** 2010. “Do expectations matter? The great moderation revisited.” *American Economic Journal: Macroeconomics*, 2(3): 183–205.
- Christiano, L.J., M. Eichenbaum, and C.L. Evans.** 2005. “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy.” *Journal of Political Economy*, 113(1).
- Cohen-Setton, Jeremie, Joshua K. Hausman, and Johannes F. Wieland.** 2017. “Supply-Side Policies in the Depression: Evidence from France.” *Journal of Money, Credit and Banking*, 49(2-3): 273–317.
- Coyle, Philip, and Taisuke Nakata.** 2019. “Optimal inflation target with expectations-driven liquidity traps.”

- Datta, Deepa D., Benjamin K. Johansson, Hannah Kwon, and Robert J. Vigfusson.** 2021. “Oil, Equities, and the Zero Lower Bound.” *American Economic Journal: Macroeconomics*, 13(2): 214–53.
- Del Negro, Marco, Raiden Hasengawa, and Frank Schorfheide.** 2016. “Dynamic Prediction Pools: An Investigation of Financial Frictions and Forecasting Performance.” *Journal of Econometrics*, 192(1): 391–405.
- Eggertsson, Gauti B.** 2011. “What fiscal policy is effective at zero interest rates?” In *NBER Macroeconomics Annual 2010, Volume 25*. 59–112. University of Chicago Press.
- Eggertsson, Gauti B., Neil R. Mehrotra, and Jacob A. Robbins.** 2019. “A Model of Secular Stagnation: Theory and Quantitative Evaluation.” *American Economic Journal: Macroeconomics*, 11(1): 1–48.
- Erceg, Christopher J, Dale W Henderson, and Andrew T Levin.** 2000. “Optimal monetary policy with staggered wage and price contracts.” *Journal of Monetary Economics*, 46(2): 281–313.
- Frederick, Shane, George Loewenstein, and Ted O’Donoghue.** 2002. “Time discounting and time preference: A critical review.” *Journal of economic literature*, 40(2): 351–401.
- Fujiwara, Shigeaki, Yuto Iwasaki, Ichiro Muto, Kenji Nishizaki, and Nao Sudo.** 2016. “Supplementary Paper Series for the “Comprehensive Assessment” (2): Developments in the Natural Rate of Interest in Japan.” Bank of Japan Review Series 16-E-12.
- Gabaix, Xavier.** 2020. “A behavioral New Keynesian model.” *American Economic Review*, 110(8): 2271–2327.
- Gali, Jordi, and Mark Gertler.** 1999. “Inflation dynamics: A structural econometric analysis.” *Journal of monetary Economics*, 44(2): 195–222.
- Geweke, John, and Gianni Amissano.** 2011. “Dynamic Prediction Pools: An Investigation of Financial Frictions and Forecasting Performance.” *Journal of Econometrics*, 164(1): 130–141.
- Gust, Christopher, Edward Herbst, David López-Salido, and Matthew E Smith.** 2017. “The empirical implications of the interest-rate lower bound.” *American Economic Review*, 107(7): 1971–2006.
- Hansen, Alvin H.** 1939. “Economic progress and declining population growth.” *American Economic Review*, 29(1): 1–15.
- Hausman, Joshua K, and Johannes F Wieland.** 2014. “Abenomics: preliminary analysis and outlook.” *Brookings Papers on Economic Activity*, 2014(1): 1–63.
- Hirose, Yasuo.** 2020. “An Estimated DSGE Model with a Deflation Steady State.” *Macroeconomic Dynamics*, 24(4): 1151–1185.

- Iiboshi, Hirokuni, Mototsugu Shintani, and Kozo Ueda.** 2022. “Estimating a Nonlinear New Keynesian Model with a Zero Lower Bound for Japan.” *Journal of Money, Credit and Banking*, 54(6): 1637–1671.
- Justiniano, Alejandro, Giorgio E Primiceri, and Andrea Tambalotti.** 2010. “Investment shocks and business cycles.” *Journal of Monetary Economics*, 57(2): 132–145.
- Kass, Robert E., and Adrian E. Raftery.** 1995. “Bayes Factors.” *Journal of the American Statistical Association*, 90(430): 773–795.
- Kuroda, Sachiko, and Isamu Yamamoto.** 2008. “Estimating Frisch labor supply elasticity in Japan.” *Journal of the Japanese and International Economies*, 22(4): 566–585.
- Lansing, Kevin J.** 2019. “Endogenous forecast switching near the zero lower bound.” *Journal of Monetary Economics*.
- Laubach, Thomas, and John C Williams.** 2003. “Measuring the natural rate of interest.” *Review of Economics and Statistics*, 85(4): 1063–1070.
- L’Huillier, Jean-Paul, Sanjay R Singh, and Donghoon Yoo.** 2023. “Incorporating diagnostic expectations into the New Keynesian framework.” *The Review of Economic Studies*.
- Lubik, Thomas A, and Frank Schorfheide.** 2004. “Testing for indeterminacy: an application to US monetary policy.” *American Economic Review*, 94(1): 190–217.
- Mertens, Karel, and Morten O Ravn.** 2014. “Fiscal policy in an expectations-driven liquidity trap.” *The Review of Economic Studies*, 81(4): 1637–1667.
- Mertens, Thomas M, and John C Williams.** 2021. “What to Expect from the Lower Bound on Interest Rates: Evidence from Derivatives Prices.” *American Economic Review* (forthcoming).
- Michaillat, Pascal, and Emmanuel Saez.** 2021. “Resolving New Keynesian Anomalies with Wealth in the Utility Function.” *The Review of Economics and Statistics*, 103(2): 197–215.
- Michau, Jean-Baptiste.** 2018. “Secular stagnation: Theory and remedies.” *Journal of Economic Theory*, 176: 552–618.
- Nakata, Taisuke.** 2017. “Uncertainty at the zero lower bound.” *American Economic Journal: Macroeconomics*, 9(3): 186–221.
- Nakata, Taisuke, and Sebastian Schmidt.** 2019. “Conservatism and liquidity traps.” *Journal of Monetary Economics*, 104: 37–47.
- Nakata, Taisuke, and Sebastian Schmidt.** 2021. “Expectations-Driven Liquidity Traps: Implications for Monetary and Fiscal Policy.” *American Economic Journal: Macroeconomics* (forthcoming).
- Ono, Yoshiyasu, and Katsunori Yamada.** 2018. “Difference or Ratio: Implications of Status Preference on Stagnation.” *Australian Economic Papers*.

- Pfäuti, Oliver, and Fabian Seyrich.** 2022. “A behavioral heterogeneous agent New Keynesian model.” UT Austin Working Paper.
- Ravn, Morten O, and Vincent Sterk.** 2021. “Macroeconomic fluctuations with HANK & SAM: An analytical approach.” *Journal of the European Economic Association*, 19(2): 1162–1202.
- Schmidt, Sebastian.** 2016. “Lack of confidence, the zero lower bound, and the virtue of fiscal rules.” *Journal of Economic Dynamics and Control*, 70: 36–53.
- Schmitt-Grohé, Stephanie, and Martín Uribe.** 2017. “Liquidity traps and jobless recoveries.” *American Economic Journal: Macroeconomics*, 9(1): 165–204.
- Schwarz, Gideon.** 1978. “Estimating the dimension of a model.” *The annals of statistics*, 461–464.
- Smets, Frank, and Rafael Wouters.** 2007. “Shocks and frictions in US business cycles: A Bayesian DSGE approach.” *American Economic Review*, 97(3): 586–606.
- Summers, Lawrence H.** 2013. “Have we Entered an Age of Secular Stagnation.” *IMF Fourteenth Annual Research Conference in Honor of Stanley Fischer, Washington, DC*, 63(1): 277–280.
- Wieland, Johannes F.** 2019. “Are negative supply shocks expansionary at the zero lower bound?” *Journal of Political Economy*, 127(3): 973–1007.
- Woodford, Michael.** 2003. *Interest and prices: Foundations of a theory of monetary policy*. Princeton University Press Princeton, NJ.

## A. PROOFS FOR SECTION 2

This section describes how we obtain the equations of the model of Section 2, as well as proofs Propositions 1, 2 and 3.

### A.1. Stationary Equilibrium

The stationary equilibrium of the baseline model is given by the following system of four equations in four stationary endogenous variables  $\{\tilde{C}_t, \tilde{Y}_t, \Pi_t, R_t\}$  for a given exogenous sequence of variables  $\{G_{z;t}, \nu_t, g_t, \delta_t\}$

$$1 = \beta E_t \left[ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\frac{1}{\sigma}} \frac{R_t}{G_{z;t+1} \Pi_{t+1}} \right] + \delta_t \tilde{C}_t \quad (\text{A.1})$$

$$(1 - \nu_t) - \omega h_t^{1-\sigma} \tilde{C}_t + \nu_t \Phi^\theta(\Pi_t) \Pi_t = \nu_t \beta E_t \left[ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\frac{1}{\sigma}} \Pi_{t+1} \Phi^\theta(\Pi_{t+1}) \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \right] \quad (\text{A.2})$$

$$R_t = \max \{1, \tilde{R}_t\} \quad (\text{A.3})$$

$$\tilde{C}_t + \tilde{G}_t = \tilde{Y}_t. \quad (\text{A.4})$$

where  $\tilde{C}_t \equiv \frac{C_t}{Z_t}$ ,  $\tilde{Y}_t \equiv \frac{Y_t}{Z_t}$ ,  $\tilde{G}_t \equiv \frac{G_t}{Z_t}$  are stationary variables and,  $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ . We choose  $\omega = (1 - \nu)g$  to normalize the full employment level of normalized output to  $\tilde{Y} = 1$ .

### A.2. Approximate equilibrium around the Permanent Liquidity Trap

When the economy is at a permanent liquidity trap, we have  $R = 1$ . We denote by  $\bar{x}$  the steady state values corresponding to the liquidity trap steady state. Variables with star denote the corresponding full-employment steady state value. Variables with hats and time-subscripts are log-deviations from the respective stationary steady state values.

$$\begin{aligned} \hat{c}_t &= \frac{\beta}{(\beta + \bar{\pi} G_z \delta \bar{c})} \hat{c}_{t+1} + \frac{\beta}{(R\beta + \bar{\pi} G_z \delta \bar{c})} \left( \hat{\pi}_{t+1} + \hat{G}_{z;t+1} \right) \\ \hat{\pi}_t &= \beta \frac{(\bar{\pi} - \pi)}{2\bar{\pi} - \pi} [ -(\hat{c}_{t+1} - \hat{c}_t) + \hat{y}_{t+1} - \hat{y}_t ] + \beta \hat{\pi}_{t+1} + \left( \frac{1 - (1 - \beta)\phi\bar{\pi}(\bar{\pi} - \pi)}{\phi\bar{\pi}(2\bar{\pi} - \pi)} \right) \hat{\nu}_t \\ &\quad + \left( \frac{(1 - \nu)g \bar{c}\bar{y}^{1-\sigma}}{\nu\phi\bar{\pi}(2\bar{\pi} - \pi)} \right) \left( \hat{c}_t + \frac{1}{\eta} \hat{y}_t \right) \\ \hat{c}_t &= \hat{y}_t - \hat{g}_t \end{aligned}$$

Collecting terms and replacing the log-linearized resource constraint we have:

$$\begin{aligned}\hat{y}_t &= \bar{\mathcal{D}}\text{E}_t(\hat{y}_{t+1} - \hat{g}_{t+1}) + \bar{\mathcal{D}}\text{E}_t(\hat{\pi}_{t+1} + \hat{G}_{z;t+1}) + \hat{g}_t \\ \hat{\pi}_t &= \tilde{\kappa} \left( \frac{\eta + 1}{\eta} \hat{y}_t - \hat{g}_t \right) + \bar{\lambda} \hat{v}_t + \bar{\varphi} \text{E}_t \Delta \hat{g}_{t+1} + \beta \text{E}_t \hat{\pi}_{t+1}\end{aligned}\tag{A.5}$$

Where  $\bar{\mathcal{D}} = \frac{1}{1 + G_z c}$ ,  $\bar{\lambda} = \left( \frac{1 - (1 - \frac{1}{2})}{2} \right)$ ,  $\tilde{\kappa} = \left( \frac{(1 - \frac{1}{2})g}{2} \frac{cy^{1+1/\eta}}{c} \right)$ , and  $\bar{\varphi} = \beta \left( \frac{1}{2} \right)$ .

We obtain equation 5 from log-linearizing the consumption Euler equation 1. It resembles the dynamic IS relationship of the standard New Keynesian model but modified by the discount coefficient  $\bar{\mathcal{D}}$ . Since  $\delta > 0$ , the discounting coefficient  $\bar{\mathcal{D}} < 1$ . Discounting dampens the consumption response to changes in the ex-ante real interest rate. An increase in the preference for bonds, lower steady-state inflation, and lower long-run growth rate increase the discounting in the Euler equation conditional on  $\delta > 0$ . We introduce shocks to growth rate of technology,  $G_{z;t}$ , to replicate movements in the real interest rate observed in Japan.

Equation 6 is the forward-looking Phillips curve that depends on expected inflation and marginal costs  $((1/\eta + 1)\hat{y}_t - \hat{g}_t)$ , the growth in government expenditure  $(\hat{g}_{t+1} - \hat{g}_t)$  and the price-markup shock  $\hat{v}_t$ . The growth in government expenditure appears in this equation because of we log-linearized the equation away from the targeted-inflation steady state.

Equation 7 is the resource constraint of the economy that specifies a time-varying wedge between consumption and output, corresponding to exogenous shocks in government spending. Equation 8 indicates that the economy operates under an interest rate peg. We can derive this equation from any policy rule in which the central bank faces an effective lower bound constraint.

### A.3. Proof of Proposition 1

Without shocks, the system of equations around a permanent liquidity trap can be rewritten as:

$$\hat{y}_t = \bar{\mathcal{D}}\text{E}_t(\hat{y}_{t+1} + \hat{\pi}_{t+1})$$

$$\hat{\pi}_t = \beta \text{E}_t \hat{\pi}_{t+1} + \tilde{\kappa} \hat{y}_t$$

Where  $\tilde{\kappa} = \frac{+1}{+} \tilde{\kappa}$ ,  $\bar{\mathcal{D}} = \frac{1}{+} \frac{1}{G_z c}$ .

In matrix form, we can write the system as:

$$\begin{bmatrix} \bar{\mathcal{D}} & \bar{\mathcal{D}} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\tilde{\kappa} & 1 \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$$

Inverting the matrix on the left hand side, we get:

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \end{bmatrix} = \begin{bmatrix} (\phi + \tilde{\kappa}\rho) & -\rho \\ -\tilde{\kappa}\rho & \rho \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix}$$

where  $\rho \equiv 1/\beta$ , and  $\phi \equiv 1/\bar{\mathcal{D}}$ . Define  $M \equiv \begin{bmatrix} (\phi + \tilde{\kappa}\rho) & -\rho \\ -\tilde{\kappa}\rho & \rho \end{bmatrix}$ . Then, we can derive the following properties of the matrix  $M$ :

$$\det(M) = \phi\rho, \quad \text{tr}(M) = \phi + (1 + \tilde{\kappa})\rho$$

Proposition C1 in (Woodford, 2003, pp 670) provides the necessary and sufficient conditions for determinacy for a system of 2 equations. A  $2 \times 2$  matrix  $M$  with positive determinant has both eigenvalues outside the unit circle if and only if

$$\det M > 1, \quad \det M - \text{tr} M > -1, \quad \det M + \text{tr} M > -1$$

Under our sign restrictions on parameters and the assumption that  $\beta < 1$ , first and third inequalities necessarily hold. It follows then that both eigenvalues are outside the unit circle if and only if  $\phi > \frac{1}{1} \frac{(1+\cdot)}{1} = 1 - \frac{1}{1}$  for determinacy. This implies  $1/\bar{\mathcal{D}} > \frac{\frac{\beta}{\beta} \frac{1}{\beta} (1+\cdot)}{\frac{\beta}{\beta} \frac{1}{\beta}} = \frac{(1)}{1} = 1 + \frac{1}{1}$ . We can rewrite this inequality to obtain  $\frac{1}{1} > \bar{\mathcal{D}}$ , which yields the restriction in the proposition.

#### A.4. Proof of Proposition 2

This section derives the unique solution for secular stagnation, given local determinacy.

Guess that  $\hat{y}_t = a_1 \hat{G}_{z,t} + a_2 \hat{g}_t + a_3 \hat{v}_t$  and  $\hat{\pi}_t = b_1 \hat{G}_{z,t} + b_2 \hat{g}_t + b_3 \hat{v}_t$  and solve for the unknown  $a^{\theta}$ s and  $b^{\theta}$ s. Replacing the guess into (A.5) and dropping time subscripts, we obtain:

$$\begin{aligned} a_1 \hat{G}_{z,t} + a_2 \hat{g}_t + a_3 \hat{v}_t &= \bar{\mathcal{D}} \left( a_1 \rho_z \hat{G}_{z,t} + a_2 \rho_g \hat{g}_t + a_3 \rho_v \hat{v}_t - \rho_g g + b_1 \rho_z \hat{G}_{z,t} + b_2 \rho_g \hat{g}_t + b_3 \rho_v \hat{v}_t + \rho_z \hat{G}_{z,t} \right) + \hat{g}_t \\ &= \bar{\mathcal{D}} (a_1 \rho_z + b_1 \rho_z + \rho_z) \hat{G}_{z,t} + (\bar{\mathcal{D}} a_2 \rho_g - \bar{\mathcal{D}} \rho_g + \bar{\mathcal{D}} b_2 \rho_g + 1) \hat{g}_t + \bar{\mathcal{D}} (b_3 \rho_v + a_3 \rho_v) \hat{v}_t \end{aligned}$$

$$\begin{aligned} b_1 \hat{G}_{z,t} + b_2 \hat{g}_t + b_3 \hat{v}_t &= \beta \left( b_1 \rho_z \hat{G}_{z,t} + b_2 \rho_g \hat{g}_t + b_3 \rho_v \hat{v}_t \right) + \bar{\varphi} (\rho_g - 1) \hat{g}_t + \bar{\lambda} \hat{v}_t \\ &+ \bar{\kappa} \left( \frac{1}{\eta} + 1 \right) \left( a_1 \hat{G}_{z,t} + a_2 \hat{g}_t + a_3 \hat{v}_t \right) - \bar{\kappa} \hat{g}_t \\ &= \left( \beta b_1 \rho_z + \bar{\kappa} \left( \frac{1}{\eta} + 1 \right) a_1 \right) \hat{G}_{z,t} + \left( \beta b_2 \rho_g + \bar{\varphi} (\rho_g - 1) + \bar{\kappa} \left( \frac{1}{\eta} + 1 \right) a_2 - \bar{\kappa} \right) \hat{g}_t \\ &+ \left( \beta b_3 \rho_v + \bar{\kappa} \left( \frac{1}{\eta} + 1 \right) a_3 + \bar{\lambda} \right) \hat{v}_t \end{aligned}$$

Comparing terms we can write the following system of equations:



$$\begin{bmatrix} 0 \\ -\bar{\varphi}(\rho_g - 1) + \bar{\kappa} \\ -\bar{\lambda} \\ -\bar{\mathcal{D}}\rho_z \\ \bar{\mathcal{D}}\rho_g - 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (\beta\rho_z - 1) & 0 & 0 & \bar{\kappa}(1 + 1) & 0 & 0 \\ 0 & (\beta\rho_g - 1) & 0 & 0 & \bar{\kappa}(1 + 1) & 0 \\ 0 & 0 & (\beta\rho_v - 1) & 0 & 0 & \bar{\kappa}(1 + 1) \\ \bar{\mathcal{D}}\rho_z & 0 & 0 & (\bar{\mathcal{D}}\rho_z - 1) & 0 & 0 \\ 0 & \bar{\mathcal{D}}\rho_g & 0 & 0 & (\bar{\mathcal{D}}\rho_g - 1) & 0 \\ 0 & 0 & \bar{\mathcal{D}}\rho_v & 0 & 0 & (\bar{\mathcal{D}}\rho_v - 1) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

The solution is:

$$b_1 = \frac{-\bar{\mathcal{D}}\rho_z \bar{\kappa} (1 + 1)}{\bar{\mathcal{D}}\rho_z [\bar{\kappa} (1 + 1) + (1 - \beta\rho_z)] - (1 - \beta\rho_z)}; \quad a_1 = \frac{(1 - \beta\rho_z)b_1}{\bar{\kappa} (1 + 1)} \quad (\text{A.6})$$

$$b_2 = \frac{(1 - \bar{\mathcal{D}}\rho_g) [\bar{\varphi}(1 - \rho_g) - \bar{\kappa}^1]}{\bar{\mathcal{D}}\rho_g [\bar{\kappa} (1 + 1)] - (1 - \beta\rho_g)(1 - \bar{\mathcal{D}}\rho_g)}; \quad a_2 = \frac{\bar{\kappa} + (1 - \beta\rho_g)b_2 + \bar{\varphi}(1 - \rho_g)}{\bar{\kappa} (1 + 1)} \quad (\text{A.7})$$

$$b_3 = \frac{-\bar{\lambda}(1 - \bar{\mathcal{D}})\rho_v}{\bar{\mathcal{D}}\rho_v [\bar{\kappa} (1 + 1) + (1 - \beta\rho_v)] - (1 - \beta\rho_v)}; \quad a_3 = \frac{\bar{\mathcal{D}}\rho_v b_3}{1 - \bar{\mathcal{D}}\rho_v} \quad (\text{A.8})$$

### A.5. Proof of Proposition 3

*Proof.* To construct the proof for correlation, first we compute various moments. Note that:

$$\mathbb{E}[\hat{y}_t] = \mathbb{E}[\hat{\pi}_t] = \mathbb{E}[\hat{G}_{z;t}] = \mathbb{E}[\hat{g}_t] = \mathbb{E}[\hat{v}_t] = 0.$$

$$\mathbb{E}[\hat{G}_{z;t}^2] = \frac{1}{1 - \rho_z^2} \sigma_z^2; \quad \mathbb{E}[\hat{g}_t^2] = \frac{1}{1 - \rho_g^2} \sigma_g^2; \quad \mathbb{E}[\hat{v}_t^2] = \frac{1}{1 - \rho^2} \sigma^2$$

From the solution  $\{a_i, b_i\} \forall i \in [1, 2, 3]$  derived in Proposition 2, we can write output growth and inflation as follows:

$$\hat{y}_t - \hat{y}_{t-1} = a_1 \epsilon_{z;t} + a_2 \epsilon_{g;t} + a_3 \epsilon_{v;t} - (1 - \rho_z) a_1 \hat{z}_{t-1} - (1 - \rho_g) a_2 \hat{g}_{t-1} - (1 - \rho) a_3 \hat{v}_{t-1}$$

$$\hat{\pi}_t = b_1 \epsilon_{z;t} + b_2 \epsilon_{g;t} + b_3 \epsilon_{v;t} + \rho_z b_1 \hat{z}_{t-1} + \rho_g b_2 \hat{g}_{t-1} + \rho b_3 \hat{v}_{t-1}$$

Correlation between output growth and inflation is then given by:

$$\rho_{d\hat{y}_t; \hat{\pi}_t} = \frac{\mathbb{E}[(\hat{y}_t - \hat{y}_{t-1}) \hat{\pi}_t]}{\sqrt{\text{Var}(\hat{y}_t - \hat{y}_{t-1}) \text{Var}(\hat{\pi}_t)}}$$

The correlation is positive if and only if the numerator is positive. Evaluating the numerator, we get:

$$\begin{aligned} \mathbb{E}[(\hat{y}_t - \hat{y}_{t-1}) \hat{\pi}_t] &= a_1 b_1 \sigma_z^2 + a_2 b_2 \sigma_g^2 + a_3 b_3 \sigma^2 \\ &\quad - ((1 - \rho_z) \rho_z a_1 b_1 \mathbb{E}[\hat{z}_{t-1}^2] + (1 - \rho_g) \rho_g a_2 b_2 \mathbb{E}[\hat{g}_{t-1}^2] + (1 - \rho) \rho a_3 b_3 \mathbb{E}[\hat{v}_{t-1}^2]) \end{aligned}$$

This can be simplified to:

$$E[(\hat{y}_t - \hat{y}_{t-1}) \hat{\pi}_t] = \frac{a_1 b_1}{1 + \rho_z} \sigma_z^2 + \frac{a_2 b_2}{1 + \rho_g} \sigma_g^2 + \frac{a_3 b_3}{1 + \rho} \sigma^2$$

From the solution  $\{a_i, b_i\} \forall i \in [1, 2, 3]$  derived in Proposition 2, note that the products  $a_1 b_1$  and  $a_3 b_3$  are non-negative. Therefore, conditional of technology growth rate shocks and price-markups shocks, inflation and output growth are (weakly) positively correlated. Positive correlation between inflation and output growth also obtains under government spending shocks if  $\bar{\kappa} < \frac{z^c (1 - \bar{\varphi})}{\eta}$ .

We can use the matrix equations to alternately rewrite output and inflation IRF to govt spending shock as follows.

$$a_2 = \frac{1 + \bar{D} \rho_g (b_2 - 1)}{1 - \bar{D} \rho_g}$$

Consequently,  $a_2 > 0$  whenever  $b_2 > 0$ .

When  $b_2 < 0$ , a condition that guarantees that  $a_2 < 0$  is  $b_2 < -\frac{1}{\bar{D} \rho_g}$  (From A.7 and the fact that  $\bar{\varphi} < 0$ ). Rewrite this condition, and substitute in the values of parameters to obtain the requirement that  $\bar{\kappa} < \frac{z^c (1 - \bar{\varphi})}{\eta}$  is sufficient for positive correlation between inflation and output growth.

This latter condition is implied by the local determinacy requirement discussed in Proposition 1. We then assumed that:

$$\frac{\tilde{\pi} G_z \delta \tilde{c}}{\beta} > \frac{1 + \eta}{\eta(1 - \beta)} \bar{\kappa}$$

which implies  $\bar{\kappa} < \frac{z^c (1 - \bar{\varphi})}{\eta}$ .

□

## A.6. Proof of Proposition 4

Before we construct the proof for Proposition 4, we describe the non-linear equations and approximate equilibrium.

Bilbiie (2021) provides a micro foundation for the static Phillips curve assumed in Section 2. Price adjustment costs are postulated in the tradition of “external” habits. Adjustment costs are such that firms consider yesterday’s market average price index instead of their own individual last-period price. That is, we assume that the adjustment costs take the following form:

$$\Phi_s \equiv \frac{\phi}{2} \left( \frac{P_t(j)}{P_t} - \Pi \right)^2 Y_t$$

The stationary equilibrium of the model with static Phillips curve is given by the following system of four equations in four stationary endogenous variables  $\{\tilde{C}_t, \tilde{Y}_t, \Pi_t, R_t\}$  for a given exogenous sequence of variables  $\{G_{z,t}, \nu_t, g_t, \delta_t\}$

$$1 = \beta E_t \left[ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{\frac{1}{\nu}} \frac{R_t}{G_{Z,t+1} \Pi_{t+1}} \right] + \delta_t \tilde{C}_t \quad (\text{A.9})$$

$$(1 - \nu) - \omega h_t^{1-\nu} \tilde{C}_t + \nu_t \Phi^\theta(\Pi_t) \Pi_t = 0 \quad (\text{A.10})$$

$$R_t = \max \{ 1, \tilde{R}_t \} \quad (\text{A.11})$$

$$\tilde{C}_t + \tilde{G}_t = \tilde{Y}_t. \quad (\text{A.12})$$

where  $\tilde{C}_t \equiv \frac{C_t}{Z_t}$ ,  $\tilde{Y}_t \equiv \frac{Y_t}{Z_t}$ ,  $\tilde{G}_t \equiv \frac{G_t}{Z_t}$  are stationary variables and,  $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ . We choose  $\omega = (1 - \nu)g$  to normalize the full employment level of normalized output to  $\tilde{Y} = 1$ .

When the economy is in a permanent liquidity trap, we have  $R = 1$ . We denote by  $\bar{x}_s$  the steady state values corresponding to the liquidity trap steady state with a static Phillips curve. Variables with a star denote the corresponding steady-state value for full employment. Variables with hats and time subscripts are log deviations from the stationary steady-state values.  $\pi_t$  is the log deviation of gross inflation from the steady state.

$$\begin{aligned} \hat{c}_t &= \frac{\beta}{(\beta + \bar{\pi}_s G_z \delta \bar{c}_s)} \hat{c}_{t+1} + \frac{\beta}{(R\beta + \bar{\pi}_s G_z \delta \bar{c}_s)} (\hat{\pi}_{t+1} + \hat{G}_{Z,t+1}) \\ \hat{\pi}_t &= \left( \frac{1 - \phi \bar{\pi}_s (\bar{\pi}_s - \pi)}{\phi \bar{\pi}_s (2\bar{\pi}_s - \pi)} \right) \hat{\nu}_t + \left( \frac{(1 - \nu)g \bar{c}_s \bar{y}_s^{1-\nu}}{\nu \phi \bar{\pi}_s (2\bar{\pi}_s - \pi)} \right) \left( \hat{c}_t + \frac{1}{\eta} \hat{y}_t \right) \\ \hat{c}_t &= \hat{y}_t - \hat{g}_t \end{aligned}$$

Collecting terms and replacing the log-linearized resource constraint, we have:

$$\begin{aligned} \hat{y}_t &= \bar{D}(\hat{y}_{t+1} - \hat{g}_{t+1}) + \bar{D}(\hat{\pi}_{t+1} + \hat{G}_{Z,t+1}) + \hat{g}_t \\ \hat{\pi}_t &= \bar{\kappa} \left( \frac{\eta + 1}{\eta} \hat{y}_t - \hat{g}_t \right) + \bar{\lambda} \hat{\nu}_t \end{aligned} \quad (\text{A.13})$$

Where  $\bar{D} = \frac{\beta}{(\beta + \bar{\pi}_s G_z \delta \bar{c}_s)}$ ,  $\bar{\lambda} = \left( \frac{1 - \phi \bar{\pi}_s (\bar{\pi}_s - \pi)}{\phi \bar{\pi}_s (2\bar{\pi}_s - \pi)} \right)$ , and  $\bar{\kappa} = \left( \frac{(1 - \nu)g \bar{c}_s \bar{y}_s^{1-\nu}}{\nu \phi \bar{\pi}_s (2\bar{\pi}_s - \pi)} \right)$ . Shutting down the government spending shocks and the TFP growth rate shocks, we obtain

$$\begin{aligned} \hat{y}_t &= \bar{D}(\hat{y}_{t+1} + \hat{\pi}_{t+1}) \\ \hat{\pi}_t &= \tilde{\kappa} \hat{y}_t + \bar{\lambda} \hat{\nu}_t \end{aligned} \quad (\text{A.14})$$

where  $\tilde{\kappa} = \bar{\kappa} \left( \frac{\eta + 1}{\eta} \right)$ . Substituting the Phillips curve into the Euler equation, we arrive at the system of log-linearized equations presented in the main text.

### A.6.1 Baseline Case: Sunspot on Inflation Forecast Error

We define the one-step ahead forecast error associated with the expectational variable  $\hat{\pi}_t$ , as:

$$\zeta_t \equiv \hat{\pi}_t - E_{t-1} \hat{\pi}_t \quad (\text{A.15})$$

Because we are analyzing the system around a locally indeterminate steady state,  $\Lambda > 1$ . We combine this equation with equation (A.15), to get the solutions of the following form:

$$\begin{aligned} \hat{\pi}_t &= \Lambda^{-1} \hat{\pi}_{t-1} - \Lambda^{-1} \bar{\lambda} \hat{v}_{t-1} + \zeta_t \\ \hat{y}_t &= \tilde{\kappa}^{-1} \hat{\pi}_t - \bar{\lambda} \tilde{\kappa}^{-1} \hat{v}_t \end{aligned}$$

In a stationary solution, the unconditional means of  $\hat{y}_t$  and  $\hat{\pi}_t$  are zero. The expression for the variance of  $\hat{\pi}_t$  is:

$$\sigma^2 = \frac{\bar{\lambda}^2}{\Lambda^2 - 1} \sigma^2 + \frac{\Lambda^2}{\Lambda^2 - 1} \sigma^2 - \frac{\lambda}{\Lambda^2 - 1} \rho ; \sigma \sigma \quad (\text{A.16})$$

where  $\sigma^2 \equiv E \zeta_t^2$ , and  $\rho ; \equiv \frac{E(\zeta_t \zeta_{t-1})}{\sigma \sigma}$ . Furthermore,

$$E[(\hat{y}_t - \hat{y}_{t-1}) \hat{\pi}_t] = \tilde{\kappa}^{-1} \left[ \frac{-\bar{\lambda}^2 \sigma^2 + \Lambda \sigma^2 - (\Lambda - 1) \bar{\lambda} \rho ; \sigma \sigma}{\Lambda + 1} \right]$$

Thus, the correlation between inflation and output growth is negative if and only if:

$$1 > \rho ; > \frac{\Lambda \sigma^2 - \bar{\lambda}^2 \sigma^2}{(\Lambda - 1) \bar{\lambda} \sigma \sigma}$$

### A.6.2 Extension: Sunspot on Output Forecast Error

We consider a variation of the baseline setup. We define the sunspot on the output forecast error instead of the inflation forecast error. We define the one-step ahead forecast error associated with output as:

$$\zeta_{y;t} \equiv \hat{y}_t - E_{t-1} \hat{y}_t$$

As above, for local indeterminacy, it follows that  $\Lambda \equiv \bar{D}(1 + \tilde{\kappa}) > 1$ . The solution to this system takes the following form:

$$\begin{aligned} \hat{y}_t &= \Lambda^{-1} \hat{y}_{t-1} + \zeta_{y;t} \\ \hat{\pi}_t &= \tilde{\kappa} \hat{y}_t + \bar{\lambda} \hat{v}_t \end{aligned}$$

In a stationary solution, the unconditional means of  $\hat{y}_t$  and  $\hat{\pi}_t$  are zero. The expression for the variance of  $\hat{y}_t$  is:

$$\sigma_y^2 = \frac{\sigma^2}{1 - \Lambda^2} = \frac{\Lambda^2}{\Lambda^2 - 1} \sigma^2 \quad (\text{A.17})$$

where  $\sigma^2 \equiv E\zeta_{y;t}^2$  with slight abuse of notation.

In order to compute correlation between inflation and output growth,

$$\begin{aligned} E[(\hat{y}_t - \hat{y}_{t-1}) \hat{\pi}_t] &= E\left([\Lambda^{-1} - 1]\hat{y}_{t-1} + \zeta_{y;t} [\Lambda^{-1}\tilde{\kappa}\hat{y}_{t-1} + \tilde{\kappa}\zeta_{y;t} + \bar{\lambda}\hat{v}_t]\right) = \frac{\tilde{\kappa}\sigma^2}{1 + \Lambda^{-1}} + \bar{\lambda}E[\nu_t\zeta_t] \\ &= \frac{\tilde{\kappa}\sigma^2}{1 + \Lambda^{-1}} + \bar{\lambda}\rho ; \sigma \sigma \end{aligned}$$

where  $\rho ; \equiv \frac{E(\nu_t \zeta_t)}{\nu \zeta}$ . The correlation of inflation with output growth is negative if and only if:

$$-1 \leq \rho ; < -\frac{\tilde{\kappa}\sigma}{(1 + \Lambda^{-1})\bar{\lambda}\sigma}$$

## B. EXTENSIONS TO STYLIZED MODEL

### B.1. Description of secular stagnation without rational expectations

We provide the equilibrium conditions for variants of the secular stagnation model with non-rational agents or heterogenous agents.

#### B.1.1 [Bordalo et al. \(2018\)](#) diagnostic agents

The non-linear equilibrium conditions are similar to those described in Appendix [A.1](#), with the main exception that we replace the rational expectations operator with the diagnostic expectations operator  $E_t$ .

The stationary equilibrium of the baseline model with diagnostic expectations is given by the following system of four equations in four stationary endogenous variables  $\{\tilde{C}_t, \tilde{Y}_t, \Pi_t, R_t\}$  for a given exogenous sequence of variables  $\{G_{z;t}, \nu_t, \tilde{G}_t, \delta_t\}$

$$1 = \beta R_t G_{z;t} \Pi_t E_t \left[ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-1} \frac{1}{G_{z;t} G_{z;t+1} \Pi_t \Pi_{t+1}} \right] + \delta_t \tilde{C}_t \quad (\text{B.1})$$

$$(1 - \nu_t) - \omega h_t^{1-\nu} \tilde{C}_t + \nu_t \Phi^\theta(\Pi_t) \Pi_t = \nu_t \beta E_t \left[ \left( \frac{\tilde{C}_{t+1}}{\tilde{C}_t} \right)^{-1} \Pi_{t+1} \Phi^\theta(\Pi_{t+1}) \frac{\tilde{Y}_{t+1}}{\tilde{Y}_t} \right] \quad (\text{B.2})$$

$$R_t = \max \{1, \tilde{R}_t\} \quad (\text{B.3})$$

$$\tilde{C}_t + \tilde{G}_t = \tilde{Y}_t. \quad (\text{B.4})$$

where  $\tilde{C}_t \equiv \frac{C_t}{Z_t}$ ,  $\tilde{Y}_t \equiv \frac{Y_t}{Z_t}$ , and  $\tilde{G}_t \equiv \frac{G_t}{Z_t}$  are stationarized variables. We choose  $\omega = (1 - \nu)g$  to normalize the full employment level of normalized output to  $\tilde{Y} = 1$ .

Following [L'Huillier et al. \(2023\)](#), we employ diagnostic expectations such that the steady state is unchanged relative to the model with rational expectations. In a linearized general equilibrium model, the diagnostic expectations operator on a future variable is defined as:

$$E_t[X_{t+1}] = (1 + \theta)E_t[X_{t+1}] - \theta E_{t-1}[X_{t+1}]; \quad \theta \geq 0$$

The log-linearized system then is given by System [B.5](#):

$$\begin{aligned} \hat{y}_t &= \bar{D}E_t(\hat{y}_{t+1} - \hat{g}_{t+1} + \hat{\pi}_{t+1} + \hat{G}_{z;t+1}) + \hat{g}_t + \theta(\hat{\pi}_t - E_{t-1}\hat{\pi}_t + \hat{G}_{z;t} - E_{t-1}\hat{G}_{z;t}) \\ \hat{\pi}_t &= \bar{\kappa}\left(\frac{\eta+1}{\eta}\hat{y}_t - \hat{g}_t\right) + \bar{\lambda}\hat{v}_t + \bar{\varphi}(E_t\hat{g}_{t+1} - \hat{g}_t) + \beta E_t\hat{\pi}_{t+1} \end{aligned} \quad (\text{B.5})$$

Where  $\bar{D} = \frac{1}{1 + \frac{1}{G_z} \frac{1}{\sigma}}$ ,  $\bar{\lambda} = \left(\frac{1 - (1 - \frac{1}{2})}{2}\right)$ ,  $\bar{\kappa} = \left(\frac{(1 - \frac{1}{2})g}{2} \frac{cy^{1+1/\eta}}{\sigma}\right)$ , and  $\bar{\varphi} = \beta \frac{1}{2}$ .

### B.1.2 [Gabaix \(2020\)](#) behavioral agents

We replace the rational expectations operator  $E_t$  in the secular stagnation equilibrium with Gabaix's bounded rationality  $E_t^{BR}$  operator. In the steady state,  $E^{BR} = E$ . In making a forecast for variable  $X_{t+k}$  which is  $k > 0$  periods in the future, the Gabaix's bounded rationality operator is linked to the rational expectations operator in the following manner:

$$E_t^{BR}[X_{t+k}] = \bar{m}^k E_t[X_{t+k}]$$

where  $\bar{m} \in [0, 1]$ .

With the exception of this change, the equilibrium conditions are same as described in System [A.5](#).

### B.1.3 [Bilbiie \(2021\)](#) heterogenous agents with behavioral expectations

We consider the case of zero steady state inequality, which is the baseline case in [Bilbiie \(2021\)](#). The focus is thus only on the cyclical inequality. This version has the advantage that we can maintain same steady state as our baseline model, introduced in Section 2.

The log-linearized Euler-equation would however be different:

$$\hat{y}_t = \mathcal{D}^T \Gamma (\hat{y}_{t+1} - \hat{g}_{t+1} + \hat{G}_{z;t+1}) + \mathcal{D}^T \frac{1 - \lambda}{1 - \lambda\chi} \hat{\pi}_{t+1} + \mathcal{D}^T \zeta \left[ \frac{\lambda(\chi - 1)}{1 - \lambda\chi} (g_t - E_t g_{t+1}) + (\Gamma - 1) E_t g_{t+1} \right] + g_t$$

where  $\mathcal{D}^T \equiv \left(-\frac{c}{1} + 1\right)^{-1}$ ,  $\Gamma \equiv 1 + (\chi - 1) \frac{1-s}{1}$ ,  $\chi \equiv (1 + \eta^{-1}) \left(1 - \frac{D}{1}\right)$ ,  $\zeta \equiv 1/(1 + \eta)$ , and  $\lambda = \frac{1-s}{2} \frac{1}{h}$ . As in [Bilbiie](#),  $\lambda$  is the unconditional mass of the hand-to-mouth agents,  $s$  and  $h$  are the probabilities that a saver and hand-to-mouth agent stay in their respective states,  $\eta$  is the Frisch elasticity of labor supply,  $\tau^D$  is the rate at which profits are taxed,  $\chi$  is a measure of cyclicality

of income inequality. We follow [Bilbiie \(2021\)](#) to set  $\chi = 1.48$ ,  $\lambda = 0.37$ , and  $1 - s = 0.04$ . Other parameters are same as in our baseline calibration. The Phillips curve is unchanged relative to the baseline:

$$\hat{\pi}_t = \bar{\kappa} \left( \frac{\eta + 1}{\eta} \hat{y}_t - \hat{g}_t \right) + \bar{\lambda} \hat{v}_t + \bar{\varphi} (\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t) + \beta \mathbb{E}_t \hat{\pi}_{t+1}$$

In addition, the behavioral THANK model augments  $\bar{m}$  in front of the rational expectations operator. We maintain [Gabaix \(2020\)](#) calibration of  $\bar{m} = 0.85$ , suggested also by [Pfäuti and Seyrich \(2022\)](#).

## C. DSGE ESTIMATION

### C.1. Data sources

We collect quarterly nominal GDP (GDP), nominal Consumption (C), nominal Investment (I), the GDP Deflator (DEF), total population (POP), hours worked (H), Earnings (W) from 1990Q1 to 2019Q4. We obtained all data from the Cabinet Office of Japan and the Ministry of Health Labor and Welfare. All data was obtained through Haver Analytics. The associated mnemonics are given in parenthesis.

**GDP:** is nominal gross domestic product (N9DP2) in billions of yen (seasonally adjusted at annual rate).

**I:** is the sum of nominal gross private domestic fixed investment (NEDFP2) and the change in private sector inventories (NEDSP2). Both series in billions of Yen, and seasonally adjusted.

**G:** is nominal government final consumption expenditure (N9GC2) in billions of Yen and is seasonally adjusted.

**C:** We collect nominal private final consumption expenditure (N9PC2), seasonally adjusted and in billions of yen.

**DEF:** is the gross domestic product deflator (JPSDEDP) seasonally adjusted and defined such that 2015 = 100.

**POP:** is the total population of 15 years old and over (FL15), in 10,000 persons. Source: Ministry of Internal Affairs and Communications.

**TH:** corresponds to the total monthly hours worked per employee in companies with 30 or more employees—all industries (JPEWTH2). **H:** corresponds to monthly scheduled hours worked per employee in companies with 30 or more employees—all industries (MHA).

**W:** Corresponds to contractual earnings index (2020=100) for companies with 30 or more employees—all industries (JPEWR2).

#### C.1.1 Data transformations

Population and the call rate are converted from monthly to quarterly data by taking quarterly averages. We construct per-capita GDP, per-capita consumption, and investment, dividing the

nominal quantities by population. We deflate per-capita variables using the GDP deflator. Lastly, we compute the quarter-on-quarter log difference of real per-capita GDP, real per-capita Consumption, and real per-capita investment and multiplied by 100. Inflation is defined as the quarter-on-quarter log difference of the GDP deflator and multiplied by 400 to convert it into annualized percentages.

We follow the same procedure detailed in Hirose (2020) to construct wages and hours worked per capita. We first construct average working days per month from the contractual hours series (H) and assume a constant eighth-hour workday. Then, we calculate the average hourly wage using total hours worked (TH) and contractual earnings (W). We compute log deviations of total daily hours worked and daily wages per hour with respect to their respective averages in our estimation sample.

## C.2. Measurement equations

### Stylized model

To match the model to the data, we construct model implied output ( $\Delta y_t^o$ ), consumption growth ( $\Delta c_t^o$ ), as quarter-on-quarter percentages, and inflation measured in annualized percentages ( $\pi_t^o$ ). We link the observed data series to the model counterparts through the following system of measurement equations:

$$\begin{aligned}\Delta y_t^o &= 100 \log(G_Z) + 100 \left( \hat{y}_t - \hat{y}_{t-1} + \hat{G}_{Z,t} \right), \\ \Delta c_t^o &= 100 \log(G_Z) + 100 \left( \hat{c}_t - \hat{c}_{t-1} + \hat{G}_{Z,t} \right), \\ \pi_t^o &= 400 \log(\bar{\pi}) + 400 \hat{\pi}_t.\end{aligned}\tag{C.1}$$

### Medium scale model

In the medium scale model, in addition to output growth, consumption growth, and inflation, we construct model implied investment growth ( $\Delta i_t^o$ ), real wage growth ( $\Delta w_t^o$ ) and, hours worked ( $l_t^o$ ). We augment measurement equations (C.1) with the following relations:

$$\begin{aligned}\Delta i_t^o &= 100 \log(G_Z) + 100 \left( \hat{i}_t - \hat{i}_{t-1} + \hat{G}_{Z,t} \right), \\ \Delta w_t^o &= 100 \log(G_Z) + 100 \left( \hat{w}_t - \hat{w}_{t-1} + \hat{G}_{Z,t} \right), \\ l_t^o &= 100 \log(\bar{l}) + 100 \hat{l}_t + \sigma_l \epsilon_t^l.\end{aligned}\tag{C.2}$$

Where  $\epsilon_t^l \sim N(0, \sigma_l^2)$  is the measurement error that helps the model fit high-frequency movements in the hours-worked series. We set the variance of the measurement error  $\sigma_l^2$  to be 10% of the sample variance in  $l_t^o$ .

## C.3. Prior distributions

Table 4 lists the priors used to estimate the DSGE model of Section 3, including information on the marginal prior distributions for the estimated parameters. Under the prior, we assume that all



estimated parameters are distributed independently, which implies that the joint prior distribution can be computed from the product of the marginal distributions.

**Table 4:** *Prior Distribution of DSGE parameters*

Parameters	Description	Distribution	P(1)	P(2)
$\rho_z$	Persistence tech. growth shock	$\mathcal{B}$	0.5	0.15
$\rho_{\nu_p}$	Persistence price markup shock	$\mathcal{B}$	0.5	0.15
$\rho_g$	Persistence gov. spending shock	$\mathcal{B}$	0.5	0.15
$\rho_\mu$	Persistence MEI shock	$\mathcal{B}$	0.5	0.15
$\rho_{\nu_w}$	Persistence wage markup shock	$\mathcal{B}$	0.5	0.15
$\rho_\eta$	Persistence risk premium shock	$\mathcal{B}$	0.5	0.15
$\sigma_z$	Std dev. tech. growth shock	$\mathcal{IG}$	0.005	Inf
$\sigma_{\nu_p}$	Std dev. price markup shock	$\mathcal{IG}$	0.005	Inf
$\sigma_g$	Std dev. gov. spending shock	$\mathcal{IG}$	0.005	Inf
$\sigma_\mu$	Std dev. MEI shock	$\mathcal{IG}$	0.005	Inf
$\sigma_{\nu_w}$	Std dev. wage markup shock	$\mathcal{IG}$	0.005	Inf
$\sigma_\eta$	Std dev. risk premium shock	$\mathcal{IG}$	0.005	Inf
$\sigma_\zeta$	Std dev. sunspot shock	$\mathcal{IG}$	0.005	Inf
$\rho(\epsilon_z, \zeta)$	Corr. sun. and tech. growth shocks	$\mathcal{U}$	0	0.5774
$\rho(\epsilon_p, \zeta)$	Corr. sun. and price markup shocks	$\mathcal{U}$	0	0.5774
$\rho(\epsilon_g, \zeta)$	Corr. sun. and gov. spending shocks	$\mathcal{U}$	0	0.5774
$\rho(\epsilon_\mu, \zeta)$	Corr. sun. and MEI shocks	$\mathcal{U}$	0	0.5774
$\rho(\epsilon_w, \zeta)$	Corr. sun. and wage markup shocks	$\mathcal{U}$	0	0.5774
$\rho(\epsilon_\eta, \zeta)$	Corr. sun. and risk premium shocks	$\mathcal{U}$	0	0.5774

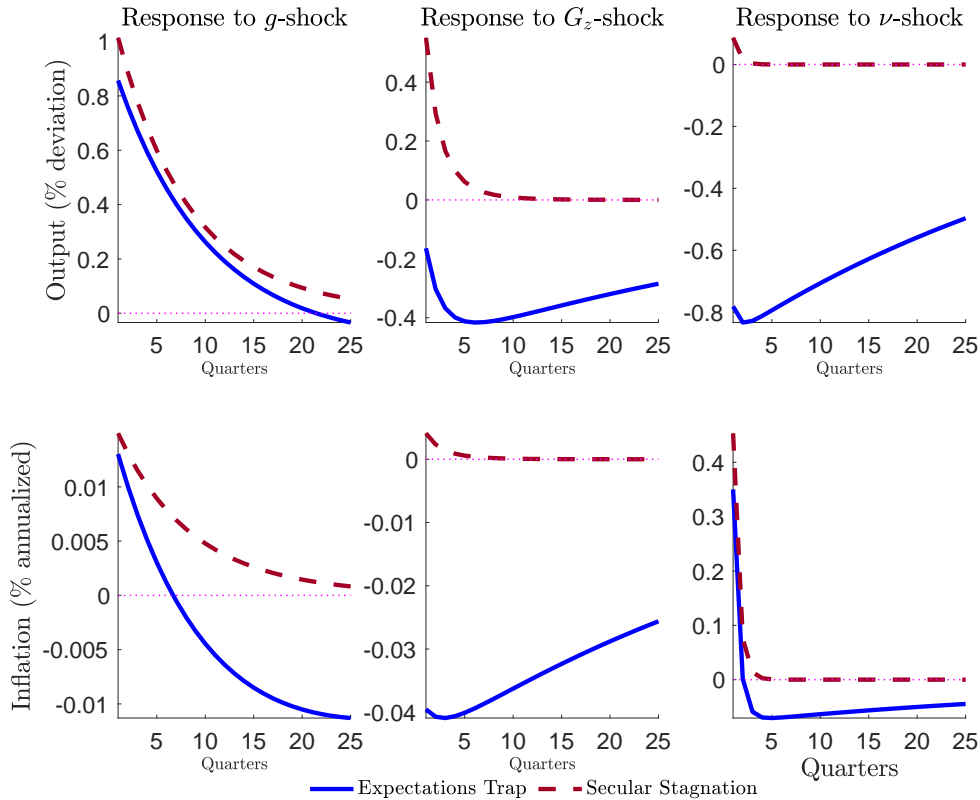
*Notes:*  $\mathcal{G}$  is Gamma distribution;  $\mathcal{B}$  is Beta distribution;  $\mathcal{IG}$  is Inverse Gamma distribution; and  $\mathcal{U}$  is Uniform distribution. P(1) and P(2) are mean and standard deviations for Beta, Gamma, and Uniform distributions.

## C.4. Posterior sampler

We can solve the log-linearized system of equations of Section 3 using standard perturbation techniques. As a result, the likelihood function can be evaluated with the Kalman filter. We generate draws from the posterior distribution using the random walk Metropolis algorithm (RWM) described in [An and Schorfheide \(2007\)](#). We scale the covariance matrix of the proposal distribution in the RWM algorithm to obtain an acceptance rate between 30% and 50%. We generated 100,000 draws from the posterior distribution and discarded the first 50,000 draws.

## C.5. Impulse Responses

**Figure 8:** *Impulse Responses: Expectations Trap vs Secular Stagnation*



*Notes:* Impulse responses to one standard deviation shocks. All responses are computed at the posterior mean of the estimated parameters. The blue solid line corresponds to the expectations-driven traps model. The red dashed line corresponds to the secular stagnation model.

## C.6. Additional Estimation Results

This section presents additional estimation results of our benchmark model. Section 4.2.2 we extend the expectations-trap model with inflation expectations data (Infl. Exp. Data). In Section 4.3 we extend the secular stagnation model to allow for diagnostic expectations (DE), cognitive discounting (Gabaix), behavioral and hand-to-mouth agents (BTHANK).

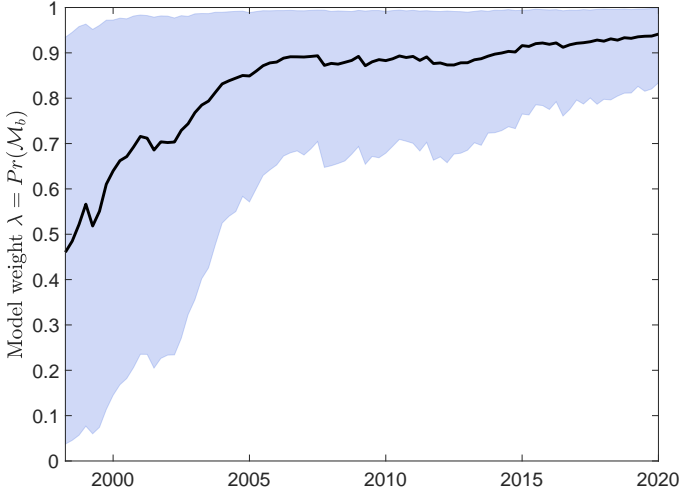
**Table 5: Posterior DSGE estimates**  
*Robustness Exercises: Baseline Model*

Parameters	Description	$\mathcal{M}_b$		$\mathcal{M}_s$	
		Infl. Exp. Data	DE	Gabaix	BTHANK
$\rho_g$	Persistence gov. spending shock	0.88 [0.83 0.92]	0.87 [0.83 0.91]	0.90 [0.86 0.95]	0.91 [0.86 0.96]
$\rho_\nu$	Persistence markup shock	0.17 [0.08 0.27]	0.15 [0.06 0.24]	0.18 [0.09 0.28]	0.12 [0.04 0.20]
$\rho_z$	Persistence technology. growth shock	0.55 [0.29 0.78]	0.47 [0.25 0.69]	0.41 [0.19 0.65]	0.47 [0.15 0.77]
$100 \times \sigma_g$	Std dev. gov. spending shock	0.94 [0.83 1.06]	0.95 [0.82 1.07]	0.92 [0.80 1.03]	0.92 [0.81 1.04]
$100 \times \sigma_\nu$	Std dev. markup shock	0.34 [0.29 0.39]	0.34 [0.26 0.41]	0.39 [0.33 0.45]	0.41 [0.35 0.47]
$100 \times \sigma_z$	Std dev. technology growth shock	0.82 [0.42 1.26]	0.42 [0.25 0.59]	0.78 [0.55 1.00]	0.78 [0.43 1.14]
$100 \times \sigma_\zeta$	Std dev. sunspot shock	0.37 [0.32 0.41]	- -	- -	- -
$\rho(\epsilon_z, \zeta)$	Corr. sunspot and technology growth shocks	-0.28 [-0.35 -0.21]	- -	- -	- -
$\rho(\epsilon_\nu, \zeta)$	Corr. sunspot and markup shocks	0.95 [0.92 0.97]	- -	- -	- -
$\rho(\epsilon_g, \zeta)$	Corr. sunspot and gov. spending shocks	0.01 [-0.02 0.05]	- -	- -	- -
$\log [p(Y^T)]$	Log-data density	-429.95	-474.58	-451.96	-460.34

*Notes:* The estimation sample is 1998:Q1 - 2012:Q4. We use  $Y^T = [y_1, \dots, y_T]$  to denote all the available data in our sample. For each model we report posterior means and 90% highest posterior density intervals in square brackets. All posterior statistics are based based on the last 50,000 draws from a RWMH algorithm, after discarding the first 50,000 draws.

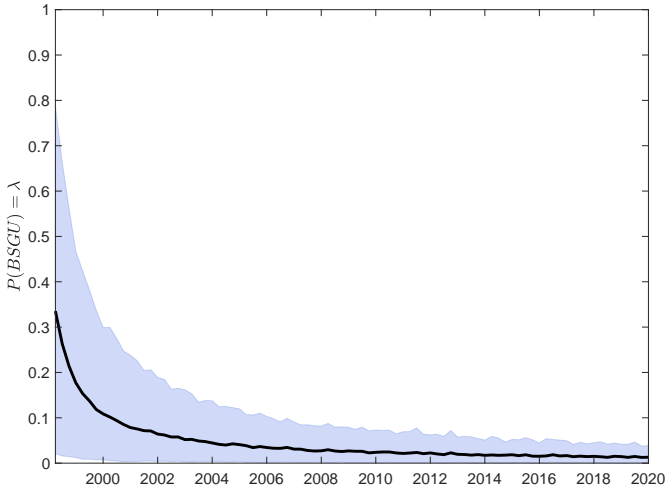
### C.7. Prediction Pools: Additional Results

**Figure 9:** *Stylized Model: Expectations-trap (w/In Exp Data) vs Secular Stagnation*



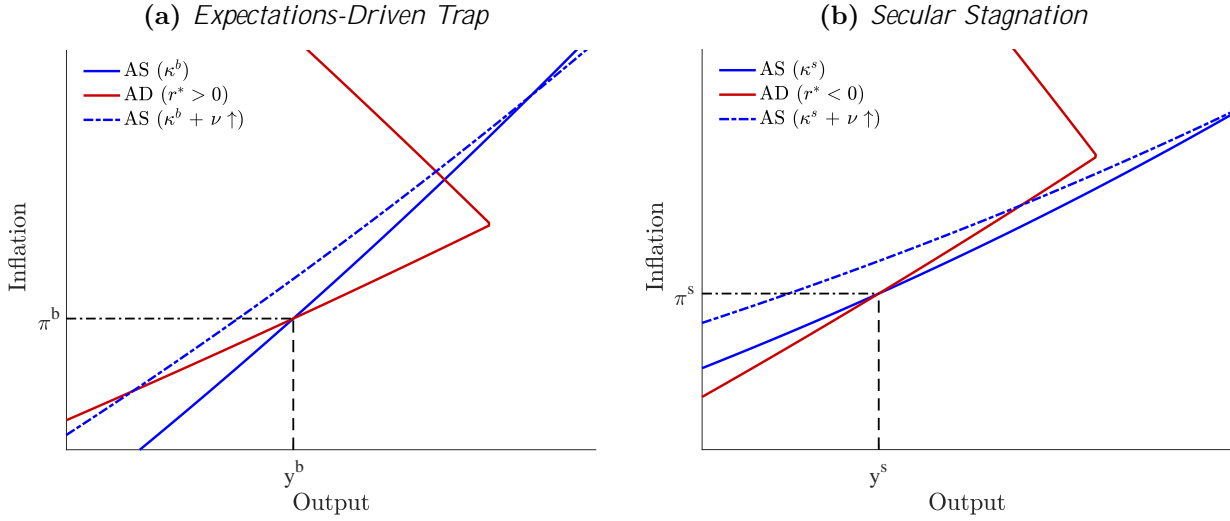
*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2019:Q4 in the stylized model of Section 3. The shaded areas correspond to the 90 percent credible set of the posterior distribution. The predictive density is constructed using three observable series:  $\Delta y_t^o, \Delta c_t^o, \pi_t^o$ .

**Figure 10:** *Medium Scale Model: Expectations-trap (w/o Markup Shocks) vs Secular Stagnation*



*Notes:* The solid black line is the posterior mean of  $\lambda$  estimated recursively over the period 1998:Q1-2019:Q4 in the medium scale model of Section 5. The shaded areas correspond to the 90 percent credible set of the posterior distribution. For the expectations-trap model we set the posterior draws of the parameters  $\rho(\epsilon_p, \zeta) = 0$  and  $\rho(\epsilon_w, \zeta) = 0$ .

**Figure 11:** *Permanent increase in markups  $\nu$*



#### D. COMPARATIVE STATICS IN THE CALIBRATED BASELINE MODEL

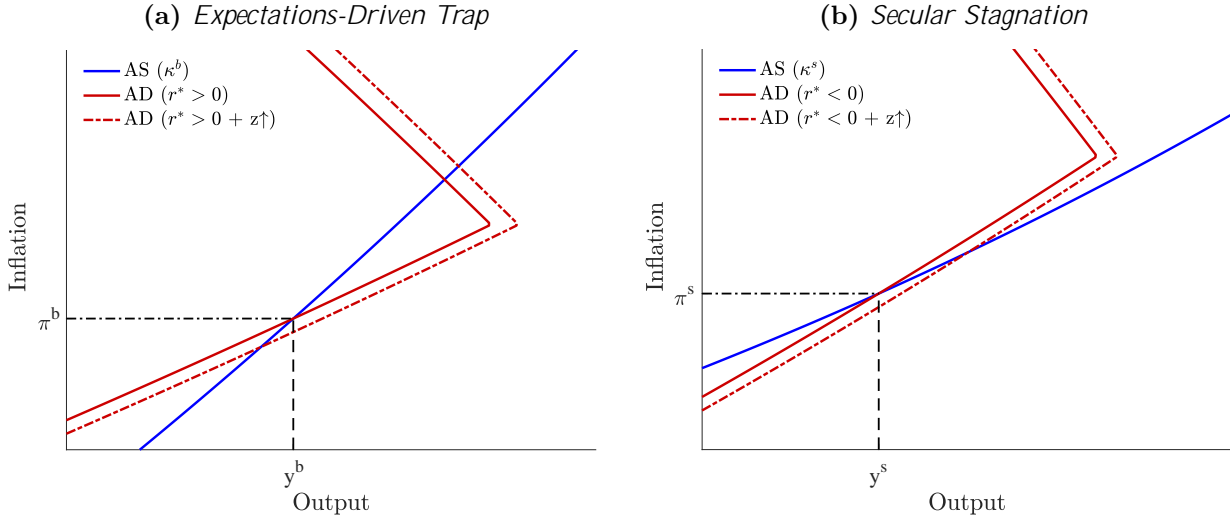
The BSGU and the secular stagnation hypotheses have contrasting implications for shocks and policy. These differences stem from the local determinacy property of these steady states, which translate into differences in slopes of aggregate supply and aggregate demand in our model. We present comparative statics in the calibrated baseline model presented in Section 2.

Because of local determinacy of the secular stagnation steady state, the comparative static experiment is well-defined without the need for additional assumptions. With the BSGU steady state, we assume that inflation expectations do not change drastically to push the economy to the full-employment steady state in response to the experiment.

In Figure 11, solid lines plot the steady-state AD-AS representation of the quantitative model under two parametrizations. Annualized inflation deviation relative to the central bank target inflation is on the vertical axes and output gap relative to target-steady state output (in percents) is on the horizontal axes. The coordinates  $(y^b, \pi^b)$  and  $(y^s, \pi^s)$  denote the expectations-driven and fundamentals-driven liquidity trap steady states respectively. The left panel plots AD-AS curves when prices are relatively flexible ( $\kappa^b$ ) and the natural rate of interest is positive. The AD-AS intersection depicted at  $(y^b, \pi^b)$  is locally indeterminate, features zero nominal interest rates and output is permanently below potential. At this intersection, the AS curve is steeper than the AD curve. In the right panel, we plot the AS curve with relatively rigid prices ( $\kappa^s$ ), and the AD curve with negative natural interest rate,  $r < 0$ . AD intersects AS at the secular stagnation steady state at the coordinate  $(y^s, \pi^s)$ .

An upward shift in aggregate supply curve in Figure 11, denoted with dashed blue line, induced by permanent increase in steady state markups, translates into higher output under secular stagnation and lower output under BSGU. Under secular stagnation, the natural interest rate is too low for the central bank to stabilize the economy. An increase in markups through inflationary pressures helps lower real interest rate, thus reducing the real interest rate gap and expand output. Under BSGU, the problem is of pessimism about inflation expectations. If agents remain pessimistic about

**Figure 12:** *Permanent increase in TFP growth rate  $z$*



inflation undershooting its target, an increase in markups is further contractionary since the resource inefficiencies associated with increased markups dominate the increase in output demand due to higher prices (see also [Mertens and Ravn, 2014](#)).

An outward shift in aggregate demand in [Figure 12](#), denoted with dashed red line, induced by permanent increase in steady state TFP growth, translates to higher output under secular stagnation but lower output under BSGU. Higher TFP growth signals higher income for households and leads to increased consumption demand. This increased impatience translates into higher output under secular stagnation. Under BSGU, in contrast, the increased TFP growth translates into higher reduction in prices by firms, which dominates the increased demand by households. As a result, there is lower output and inflation under BSGU.

Similarly, a neo-Fisherian exit policy of raising interest rates at the ZLB is contractionary under secular stagnation as it increases the real interest rate gap from natural rate, but it is expansionary at the BSGU steady state equilibrium ([Schmitt-Grohé and Uribe, 2017](#)). Furthermore, an increase in government expenditure (financed by lumpsum taxes) or a permanent reduction in short-term interest rates below the ZLB has inflationary effects under secular stagnation but deflationary effects under BSGU.<sup>26</sup>

These disparate policy implications raise the question whether it is possible to distinguish these two different kinds of liquidity traps in the data.

<sup>26</sup>We model the neo-Fisherian policy as a permanent change in the intercept of the Taylor rule,  $a$ :  $R^{new} = \max\{1 + a, a + R^* (-)^{\phi_\pi}\} = a + R$ . where  $a$  is increased to a positive number from zero. This policy simultaneously increases the lower bound on nominal interest rate and thus does not have any effect on the placement of the kink in the aggregate demand curve. Given the inflation rate, an increase in  $a$  lowers output demanded. At the secular stagnation steady state, this induces deflationary pressures that increases the real interest rate gap and causes a further drop in output. In contrast, during a BSGU trap, an increase in nominal interest rate anchors agents' expectations to higher levels of inflation, thus obtaining the neo-Fisherian results ([Schmitt-Grohé and Uribe, 2017](#)). The effects of increased government spending on output are somewhat ambiguous because of elastic labor supply that also causes changes in the aggregate supply curve.

## E. STEADY STATE ANALYSIS WITH LINEAR PHILLIPS CURVE

With the help of some simplifying assumptions in the derivation of the new Keynesian Phillips curve, we can analytically characterize the steady states in the baseline model. All the ingredients of the model are same as studied in Section 2 except we assume that the cost of price adjustment takes the following functional form:

$$\Phi_s \equiv \frac{\phi}{2} \frac{\left( \frac{P_t(j)}{P_t} - \Pi \right)^2}{\Pi_t} Y_t$$

This adjustment cost function combines the insights of Bilbiie (2021) to make the Phillips curve static, along with the insights of Bhattacharai, Eggertsson and Gafarov (2022) which deliver an aggregate Phillips curve that is linear. Under the assumption of infinitely elastic labor supply ( $\eta \rightarrow \infty$ ), and adjustment cost function of the form  $\Phi_s$ , we get the following static aggregate Phillips curve relationship between aggregate inflation rate  $\Pi_t$  and stationary consumption  $\tilde{C}_t$ :<sup>27</sup>

$$\Pi_t = \frac{1 - \nu}{\phi \nu_t} \tilde{C}_t + \left( \Pi - \frac{1 - \nu_t}{\phi \nu_t} \right) \quad (\text{E.1})$$

This static Phillips curve is the Rotemberg costs equivalent of the analytical Phillips curve derived by Bhattacharai et al. (2022). Other than that, the model is same as the baseline model.

We assume there is no government spending in the steady state for now (i.e.  $g = 1$ ). From equations 1, 3, 4, and E.1 we can represent the steady-state equilibrium with an aggregate demand block and an aggregate supply block. We describe each of these blocks next.

**Aggregate Demand (AD)** is a relation between output and inflation and is derived by combining the Euler equation and the Taylor rule. The AD curve is given by

$$\tilde{Y}_{AD} = \frac{1}{\delta} \begin{cases} 1 - \beta r \left( \frac{1}{\pi} \right)^{\pi - 1}, & \text{if } R > 1, \\ 1 - \frac{1}{\pi}, & \text{if } R = 1. \end{cases} \quad (\text{E.2})$$

When the ZLB is not binding, the AD curve has a strictly negative slope; and it becomes linear and upward sloping when the ZLB constrains the nominal interest rate. Thus, the kink in the aggregate demand curve occurs at the inflation rate where the ZLB constrains monetary policy:  $\Pi_{kink} = \left( \frac{1}{r} \right)^{\frac{1}{\phi \pi}} \Pi$ . For the natural interest rate to be positive  $r > 1$ , the premium on government bonds must be low enough i.e.  $\delta < 1 - \beta$ .

**Aggregate Supply (AS)** is given by  $\Pi = \kappa \tilde{Y} + (\Pi - \kappa)$  in the steady state, where  $\kappa \equiv \frac{1}{h}$ . When  $h = 1$ ,  $\Pi = \Pi$ . In this linear aggregate supply curve, the degree of nominal rigidity  $\kappa$  also determines a lower bound on the inflation  $\Pi = \Pi - \kappa$ .

In this two-equation representation, we can characterize different steady-state equilibria. Proposition 5 shows that a targeted steady state exists as long as the natural interest rate is positive.

---

<sup>27</sup>Note that  $\omega$  is set equal to  $(1 - \nu)$  in the steady state to target  $h = 1$ .

**Proposition 5.** (Targeted Steady State): Let  $\Pi = 1$ ,  $0 < \delta < 1 - \beta$ . There exists a unique positive interest rate steady state with output at potential  $\tilde{Y} = 1$ , inflation at target  $\Pi = 1$  and  $R = \frac{1}{1-\beta} > 1$ . It features output at efficient steady state, and inflation at the policy target. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

*Proof.* The downward sloping portion of aggregate demand always goes through  $Y = 1$  and  $\Pi = 1$  when  $\Pi = 1$ . When  $\delta < 1 - \beta$ ,  $r > 1$ . The kink in the AD curve occurs at inflation rate below 1. There always exists an intersection between the AS and the AD at  $Y = 1, \Pi = 1$ . To show that there does not exist another steady state at positive interest rates, note the AS curve is linear and upward sloping. For  $\Pi > 1$ ,  $Y_{AD} < 1 < Y_{AS}$ , and for  $\Pi_{kink} \leq \Pi < 1$ ,  $Y_{AD} > 1 > Y_{AS}$ . Thus, there does not exist another steady state at positive nominal interest rate.

To prove local determinacy, log-linearize the equilibrium conditions around the unique non-stochastic steady state, and it follows that the Blanchard-Kahn conditions for determinacy are satisfied as  $\phi > 1$ .  $\square$

A steady state at which the central bank can meet its inflation target is *de ned* as the *targeted-inflation steady state*. The presence of a targeted-inflation steady state is contingent on the natural interest rate and the monetary authority's inflation target. With a unitary inflation target, it must be the case that the natural interest rate is non-negative, which is implied by the assumption of  $0 < \delta < 1 - \beta$ . In Proposition 6 we show that, a liquidity trap steady state (à la Schmitt-Grohé and Uribe, 2017) may jointly co-exist with the targeted steady state described above. However, with a flat enough Phillips curve, a targeted steady state is the unique steady state in this economy. A high enough nominal rigidity prevents inflation from falling to levels such that self-fulfilling deflationary expectations do not manifest in the steady state.

**Proposition 6.** (Deflationary expectations-trap steady state): Let  $\Pi = 1$ ,  $0 < \delta < 1 - \beta$ . For  $\kappa > 1 - \beta$  (i.e.  $\phi < \frac{1}{1-\beta}$ ) there exist two steady states:

1. The targeted steady state with output at potential  $\tilde{Y} = 1$ , inflation at target  $\Pi = 1$  and positive nominal interest rate  $R = \frac{1}{1-\beta} > 1$ .
2. (*Deflationary expectations-driven trap*) A unique-ZLB steady state with output below potential  $\tilde{Y} < 1$ , inflation below target  $\Pi < 1$  and zero nominal interest rate  $R = 1$ . The local dynamics in a neighborhood around this steady state are locally indeterminate.

When prices are rigid enough, i.e.,  $\kappa < 1 - \beta$ , there exists a unique steady state, and it is the targeted inflation steady state. When prices are flexible  $\phi = 0$  ( $\kappa \rightarrow \infty$ ), two steady states exist. A unique deflationary steady state with zero nominal interest rates and a unique targeted inflation steady state.

*Proof.* When  $\delta < 1 - \beta$ ,  $R > 1$ . The kink in the AD occurs at inflation rate below 1. Following the steps in Proposition 5, it follows that a targeted steady state with output at potential, inflation at target, and positive nominal interest rate exists. At  $\Pi_{kink}$ ,  $Y_{AD} > 1 > Y_{AS}$ . When  $\Pi = 1 - \kappa$ ,  $Y_{AS} = 0 > Y_{AD}$ . Given that AS is linear and AD is strictly convex, there is a unique intersection at positive unemployment. At this second steady state, AS intersects AD from above. In the model, the relative steepness of AS curve is equivalent to locally indeterminate dynamics.  $\square$

We define the *deflationary expectations-driven trap* as the steady state with a positive natural interest rate, negative output gap, and deflation and in whose neighborhood the equilibrium dynamics are *locally indeterminate*. Pessimistic inflationary expectations can push the economy to this steady state without any change in fundamentals.

We now consider the case where adverse fundamentals can push the economy to a permanent liquidity trap. If agents are sufficiently patient  $\delta > \frac{1}{1-\beta}$ , i.e., the natural rate of interest is negative,



and the ZLB constrains monetary policy. In that case, the nominal interest rate is permanently zero while there is below-potential output and deflation in the economy. We characterize this possibility in Proposition 7.

**Proposition 7.** (Secular Stagnation): Let  $\Pi = 1$ ,  $\delta > 1 - \beta$  and  $\kappa < 1 - \beta$ . There exists a unique steady state with output below potential  $\tilde{Y} < 1$ , inflation below target  $\Pi < 1$  and zero nominal interest rate  $R = 1$ . It features output below the targeted steady state and deflation, caused by a permanently negative natural interest rate. The equilibrium dynamics in this steady state's neighborhood are locally determinate.

*Proof.* When  $\delta > 1 - \beta$ ,  $R < 1$ . Thus, the kink in the AD occurs at inflation rate above 1. AS is linear and AD is strictly convex. At  $\Pi_{kink}$ ,  $Y_{AD} < 1 < Y_{AS}$ . The last inequality requires the assumption that  $\kappa < 1 - \beta$ . Thus, the AS and AD must intersect at positive unemployment. With log-linearized equilibrium conditions, it can be seen that the system is locally determinate. In this case, the local determinacy condition is equivalent to the AD curve being steeper than the AS curve at the stagnation steady state. □

We formally *define* the *secular stagnation steady state* as the steady state featuring negative output gap, zero nominal interest rate on short-term government bonds and exhibiting *locally determinate* equilibrium dynamics in its neighborhood. This local determinacy property is the main difference between the secular stagnation narrative and the expectations-driven narrative.

Note that the secular stagnation steady state exists in this model because of sufficient discounting in the modified Euler equation. Unlike the traditional new Keynesian model, an arbitrarily long ZLB episode driven by a negative natural rate can exist in equilibrium. In log-linearized new Keynesian models without discounting, *stationary black holes* emerge as the duration of the temporary liquidity trap is increased, with inflation and output tending to negative infinity (Eggertsson, 2011). The solution remains bounded in our setup as the duration of ZLB is increased.

## F. MEDIUM-SCALE DSGE MODEL

Here we provide a detailed derivation of the model in Section 5. The exposition follows [Gust, Herbst, López-Salido and Smith \(2017\)](#). There are five agents in the economy: (i) monopolistically competitive intermediate goods firms (ii) a perfectly competitive firm that aggregates the differentiated varieties from intermediate producers; (iii) a perfectly competitive employment agency that bundles households' labor services (iv) a continuum of households that make optimize consumption, investment, capital utilization, and supply differentiated labor services to a labor agency; (v) the government that sets fiscal and monetary policy.

### F.1. Model Description

#### Intermediate and final goods firms

Intermediate goods producers sell the intermediate varieties  $Y_{jt}$  to the final good firms that produce the final composite good:  $Y_t = \left[ \int_0^1 Y_{jt}^{1-\nu_{p,t}} dj \right]^{\frac{1}{\nu_{p,t}}}$ , where  $1/\nu_{p,t} > 1$  is a time-varying price-markup  $\lambda_t^p = \frac{1}{1-\nu_{p,t}}$ . The demand for intermediate good  $j$  has the iso-elastic form  $Y_{jt} = \left( \frac{P_{jt}}{P_t} \right)^{-\lambda_{p,t}^p} Y_t$  where  $P_{jt}$  is the price of variety  $j$  and  $P_t$  is the aggregate price index. Each intermediate good  $j$  is produced by a price-setting monopolist using the following technology:  $Y_{jt} = (Z_t L_{jt})^\alpha K_{jt}^{1-\alpha}$ . Where  $L_{jt}$  is the labor input,  $K_{jt}$  is physical capital, and  $Z_t$  denotes a non-stationary TFP process. The growth rate of  $Z_t$  denoted by  $G_{Z,t}$  follows an AR(1) process with persistence  $\rho_Z$  and iid shocks  $\epsilon_{Z,t} \sim iid N(0, \sigma_Z^2)$  that causes deviations of the TFP growth from balanced growth rate  $G_Z$ .

Firms choose inputs to minimize total cost each period. Cost minimization implies that the capital-labor ratio at the firm level is independent of firm-specific variables  $\frac{K_{jt}}{L_{jt}} = \frac{K_t}{L_t} = \frac{1}{\alpha} \frac{W_t}{R_t}$ . As a result, nominal marginal costs are  $P_t mc_t = \frac{1}{Z_t^{1-\alpha}} \left( \frac{R_t}{W_t} \right)^\alpha \left( \frac{W_t}{R_t} \right)^{1-\alpha}$ . To set the price  $P_{jt}$ , intermediate firm  $j$  pays a quadratic adjustment cost in units of final good  $\frac{\phi_p}{2} \left( \frac{P_{jt}}{P_{j,t-1}} - 1 \right)^2 P_t Y_t$ . Where  $\phi_p \geq 0$  is the parameter that scales the cost of price changes and price adjustments are indexed to  $\tilde{\Pi}_t = \Pi_t^{1-\nu_p} \bar{\Pi}^{\nu_p}$ , where  $\nu_p$  governs indexation between previous period inflation rate  $\Pi_{t-1}$  and steady-state inflation rate  $\bar{\Pi}$ . Firm's per period profits are given by:  $\Gamma_t \equiv P_{jt} Y_{jt} - P_t mc_t N_{jt} - \frac{\phi_p}{2} \left( \frac{P_{jt}}{P_{j,t-1}} - 1 \right)^2 P_t Y_t$ . And the firm's profit maximization problem is

$$\max_{P_{jt}} \left\{ \Gamma_t + E_t \sum_{s=1}^{\infty} Q_{t,t+s} \Gamma_{t+s} \right\} \quad (\text{F.1})$$

where  $Q_{t,t+s}$  is the nominal stochastic discount factor of the household.

#### Households

A continuum of households, indexed by  $i \in [0, 1]$ , supply differentiated labor services  $L_{it}$  to a perfectly competitive labor agency. The agency combines labor services into a homogeneous labor composite  $L_t$  according to  $L_t = \left[ \int_0^1 L_{it}^{1-\nu_{w,t}} di \right]^{\frac{1}{\nu_{w,t}}}$ , where  $1/\nu_{w,t} > 1$  is the elasticity of demand that

determines the time-varying wage-markup  $\lambda_t^W = \frac{1}{1 - \frac{1}{w_t}}$ . The demand for labor inputs of type  $j$  is  $L_{j;t} = \left(\frac{W_{j;t}}{W_t}\right)^{1 - \frac{1}{w_t}} L_t$ , where  $W_{j;t}$  is the wage set by the union on behalf of the workers, and  $W_t$  is the aggregate nominal wage. At time  $t$ , the household- $i$  chooses consumption  $C_{i;t}$ , risk-free nominal bonds  $B_t$ , investment  $I_t$  and capital utilization  $u_t$  to maximize the utility function, with external habits in consumption:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log(C_{j;s} - h C_{j;s-1}) - \frac{\omega}{1 + 1/\eta} L_{j;s}^{1+1/\eta} + \delta_t \frac{B_{t+1}}{Z_t P_t} - \psi_{j;s}^W \right], \quad (\text{F.2})$$

where  $h$  is the degree of habit formation on internal habits over individual consumption.  $\eta > 0$  is the Frisch elasticity of labor supply,  $\omega > 0$  is a parameter that pins down the steady-state level of hours, exogenous parameter  $\delta_t$  regulates the utility from bonds,<sup>28</sup> and the discount factor  $\beta$  satisfies  $0 < \beta < 1$ . The utility loss,  $\psi_{j;t}^W$ , due to adjustments in the nominal wage takes the form  $\psi_{j;t}^W = \frac{w}{2} \left[ \frac{W_{j;t}}{w_{j,t-1} W_{j,t-1}} \right]^2$ . Where  $\phi_w \geq 0$  is a parameter, wage contracts are indexed to productivity and price inflation. We assume  $\tilde{\Pi}_t^W = G_Z \bar{\Pi}^{1 - \omega} (\exp(\epsilon_{Z,t}) \Pi_t)^{-\omega}$  with  $0 \leq \omega < 1$ . We assume perfect consumption risk sharing across the households. As a result, the household's budget constraint in period  $t$  is given by

$$P_t C_{i;t} + P_t I_{i;t} + \frac{B_{i;t+1}}{(1 + i_t)} = B_{i;t} + B_{i;t}^S + W_t L_{i;t} + \Gamma_t + T_t + R_t^K u_{i;t} K_t^U - P_t a(u_{i;t}) K_{i;t}^U, \quad (\text{F.3})$$

where  $I_{i;t}$  is investment,  $B_{i;t}^S$  is the net cash flow from household  $i$ 's portfolio of state-contingent securities. Households own an equal share of all firms and thus receive  $\Gamma_t$  dividends from profits. Finally, each household receives a lump-sum government transfer  $T_t$ . Since households own capital, they choose the utilization rate. The amount of effective capital that the households rent to the firms at the nominal rate  $R_t^K$  is  $K_{i;t} = u_t K_{i;t}^U$ . The unit nominal cost of capital utilization is  $P_t a(u_{i;t})$ . As in the literature (Smets and Wouters, 2007) we assume  $a(1) = 0$ , and the elasticity of utilization costs is parameterized by  $a''(1) > 0$ . Following Christiano et al. (2005), we assume investment adjustment costs in the production of capital. The law of motion for capital is as follows:

$$K_{i;t+1}^U = \mu_t \left[ 1 - S \left( \frac{I_{i;t}}{G_Z I_{i;t-1}} \right) \right] I_{i;t} + (1 - \delta_k) K_{i;t}^U \quad (\text{F.4})$$

Where  $G_Z$  is the steady state growth rate of  $Z_t$ ,  $\delta_k$  is the depreciation rate of capital, and  $\mu_t$  is an exogenous disturbance to the marginal efficiency of investment. We assume the presence of investment adjustment costs that satisfy  $S(1) = S'(1) = 0$ , and the elasticity of adjustment costs to investment changes is given by  $S''(1) > 0$ .

## Fiscal and Monetary Policy

We assume the government balances the budget every period  $P_t T_t = P_t G_t$  and  $G_t$  is the government spending, which is determined exogenously as a fraction of GDP  $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ , where  $g_t$  is an exogenous shock to government spending.

<sup>28</sup>We assume that  $\delta_t$  evolves as an AR(1) process, labeled as  $\eta_t$  later in the model.

The central bank sets the nominal interest rate  $i_t$ , following an inertial rule that responds to deviations of inflation from a constant target  $\bar{\Pi}$ , and output growth relative to the economy's long-run growth rate.

$$\frac{1+i_t}{1+i_{SS}} = \max \left( \frac{1}{1+i_{SS}}, \left( \frac{1+i_{t-1}}{1+i_{SS}} \right)^{\rho_R} \left[ \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\pi} \left( \frac{Y_t}{G_Z Y_{t-1}} \right)^{\alpha} \right]^{\rho_R} \right), \quad (\text{F.5})$$

where  $i_{SS}$  is the steady state nominal interest rate,  $\Pi_t$  is the gross inflation rate,  $\rho_R$  determines the degree of inertia in interest rate changes. The first term inside the max operator captures the effective lower bound on the nominal interest rate. Throughout our analysis, we will assume that such a lower bound will be binding.

## Market clearing

We focus on a symmetric equilibrium that satisfies  $K_t = \int_0^1 K_{i;t} di$ ,  $N_t = \int_0^1 L_{i;t} di = \int_0^1 L_{j;t} dj$ . In this symmetric equilibrium, the market clearing for the final good requires

$$Y_t = C_t + I_t + a(u_t)K_t^u + G_t + \frac{\phi\rho}{2} \left[ \frac{\Pi_t}{\bar{\Pi}_{t-1}} - 1 \right]^2 Y_t \quad (\text{F.6})$$

## F.2. Equilibrium Conditions

We present the model's equilibrium conditions as stationary variables. Let  $Z_t$  be the non-stationary level of TFP at time  $t$ . We normalize the following variables :

$$\begin{aligned} y_t &= Y_t/Z_t, \\ c_t &= C_t/Z_t, \\ k_t &= K_t/Z_t, \\ k_t^u &= K_t^u/Z_t, \\ l_t &= I_t/Z_t, \\ w_t &= W_t/(Z_t P_t), \\ r_t^k &= R_t^k/P_t, \\ \lambda_t &= \Lambda_t Z_t, \end{aligned}$$

**Definition 1** (Normalized equilibrium). 17 endogenous variables  $\{\lambda_t, i_t, c_t, y_t, \Pi_t, mc_t, \bar{\Pi}_{t-1}, \Pi_t^w, \bar{\Pi}_{t-1}^w, w_t, L_t, k_{t+1}^u, r_t^k, l_t, q_t, u_t, k_t\}$ , 6 endogenous shock processes  $\{z_t, g_t, \eta_t, \mu_t, \nu_{p;t}, \nu_{w;t}\}$ , 6 exogenous shocks  $\{\epsilon_{z;t}, \epsilon_{g;t}, \epsilon_{\eta;t}, \epsilon_{\mu;t}, \epsilon_{\nu_p;t}, \epsilon_{\nu_w;t}\}$  given initial values of  $k_t^u$ .

## Consumption Euler equation

$$\lambda_t = \beta(1+i_t)E_t \left[ \frac{\lambda_{t+1}}{G_{Z;t+1} \Pi_{t+1}} \right] + \delta_t, \quad (\text{F.7})$$

$$\lambda_t = \frac{1}{c_t - \frac{hc_{t-1}}{G_{Z,t}}} - h\beta E_t \frac{1}{G_{Z;t+1} \left[ c_{t+1} - \frac{hc_t}{G_{Z,t+1}} \right]}, \quad (\text{F.8})$$

### Price-setting

$$(1 - \nu_{p;t}) - mc_t + \nu_{p;t} \phi_p \left( \frac{\Pi_t}{\tilde{\Pi}_t} - 1 \right) \frac{\Pi_t}{\tilde{\Pi}_t} - \nu_{p;t} \phi_p \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{\Pi_{t+1}}{\tilde{\Pi}_t} - 1 \right) \frac{\Pi_{t+1}}{\tilde{\Pi}_t} \frac{y_{t+1}}{y_t} = 0 \quad (\text{F.9})$$

$$\tilde{\Pi}_t = \bar{\Pi}^1 - \nu_{p;t} \Pi_t^p \quad (\text{F.10})$$

### Wage-setting

$$\nu_{w;t} \phi_w \left[ \frac{\Pi_t^w}{\tilde{\Pi}_t^w} - 1 \right] \frac{\Pi_t^w}{\tilde{\Pi}_t^w} = \nu_{w;t} \phi_w \beta E_t \left[ \frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} - 1 \right] \frac{\Pi_{t+1}^w}{\tilde{\Pi}_t^w} + L_t \lambda_t \left[ \omega \frac{L_t^{\frac{1}{\eta}}}{\lambda_t} - (1 - \nu_{w;t}) w_t \right] \quad (\text{F.11})$$

$$\tilde{\Pi}_t^w = G_Z \bar{\Pi}^1 - \nu_{w;t} \left( \exp(\hat{G}_{Z;t}) \Pi_t \right)^w \quad (\text{F.12})$$

$$\Pi_{w;t} = \frac{w_t}{w_{t-1}} \Pi_t G_{Z;t}, \quad (\text{F.13})$$

### Capital investment

$$k_{t+1}^u = \mu_t \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \frac{G_{Z;t}}{G_Z} \right) \right] l_t + (1 - \delta_k) \frac{k_t^u}{G_{Z;t}}, \quad (\text{F.14})$$

$$q_t = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t G_{Z;t+1}} (r_{t+1}^K u_{t+1} - a(u_{t+1}) + q_{t+1} (1 - \delta_k)) \right], \quad (\text{F.15})$$

$$q_t \mu_t \left[ 1 - S \left( \frac{l_t}{l_{t-1}} \frac{G_{Z;t}}{G_Z} \right) - S^\theta \left( \frac{l_t}{l_{t-1}} \frac{G_{Z;t}}{G_Z} \right) \frac{l_t}{l_{t-1}} \frac{G_{Z;t}}{G_Z} \right] + \beta E_t \left[ \mu_{t+1} \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \frac{G_{Z;t+1}}{G_Z} \left( \frac{l_{t+1}}{l_t} \right)^2 S^\theta \left( \frac{l_{t+1}}{l_t} \frac{G_{Z;t+1}}{G_Z} \right) \right] = 1 \quad (\text{F.16})$$

### Capital utilization rate

$$k_t = u_t \frac{k_t^u}{G_{Z;t}}, \quad (\text{F.17})$$

$$r_t^K = a^\theta(u_t), \quad (\text{F.18})$$

### Production technologies

$$y_t = k_t L_t^1, \quad (\text{F.19})$$

$$r_t^K = \alpha mc_t \frac{y_t}{k_t}, \quad (\text{F.20})$$

$$w_t = (1 - \alpha) mc_t \frac{y_t}{L_t}, \quad (\text{F.21})$$

## Government

$$\frac{1+i_t}{1+i_{ss}} = \max \left( \frac{1}{1+i_{ss}}, \left( \frac{1+i_{t-1}}{1+i_{ss}} \right)^R \left[ \left( \frac{\Pi_t}{\bar{\Pi}} \right)^\pi \left( \frac{y_t G_{Z;t}}{y_{t-1} G_Z} \right)^{dy} \right]^{1-R} \exp(\epsilon_{mp;t}) \right), \quad (\text{F.22})$$

## Market clearing

$$y_t = c_t + l_t + a(u_t) \frac{k_t^u}{G_{Z;t}} + \left( 1 - \frac{1}{g_t} \right) y_t, \quad (\text{F.23})$$

**Law of Motion of Shocks** The six structural shocks driving the model economy are assumed to follow first-order auto-regressive processes of the form  $\log(x_t) = (1 - \rho_x) \log(x) + \rho_x \log(x_{t-1}) + \sigma_x \epsilon_{x;t}$ , with  $\epsilon_{x;t} \sim N(0, 1)$ , and  $x$  denoting steady-state values, for  $x = G_Z, g, \eta, \mu, \nu_p, \nu_w$ .

## F.3. Parameters and Model Calibration

Table 6 details the parameters for the medium-scale DSGE model of section 5 that are fixed before estimation in both the secular stagnation and expectations-trap models. We source most of the parameters from the estimated DSGE model for Japan in Hirose (2020). Our medium-scale model shares many features with the estimated model in Hirose (2020). We rely on the existing estimates of certain structural parameters to focus our empirical assessment on the equilibrium dynamics implied by our alternative stagnation hypothesis. The average growth rate of productivity,  $G_Z$ , is set to match the average growth rate of GDP from 1998:Q1-2012:Q4. The steady-state value of the government spending shock,  $g$ , is set to match the share of the autonomous spending-to-GDP ratio during the same period. In the medium-scale model, autonomous spending includes nominal government spending and net exports.

Lastly, Table 6 also shows the value of the parameters calibrated for each model independently. We also show the respective calibration targets. As discussed in Section 2, the marginal utility of bond holdings,  $\delta$ , pins the natural rate of interest in our model. The parameter  $\psi_p$ , which controls the cost of adjusting prices, is set such that the prices are twice as flexible in the secular stagnation model compared with the expectations-trap model. Given the natural rate and the cost of price adjustment, the parameter  $\psi_w$ , which controls the cost of adjusting wages, is endogenously calibrated to match a steady inflation of  $-1.06$  observed in our sample. To keep both secular stagnation and expectation traps on equal footing, we set the disutility of hour worked  $\omega$  such that in the steady state with a binding zero lower bound, hours worked are below steady state hours worked under full employment.

**Table 6: Structural Parameters: Medium Scale Model**

Common Parameters					
$\beta$	$\alpha$	$\gamma$	$1/(1 - \nu_p)$	$1/(1 - \nu_w)$	$\gamma_p$
Discount factor	Capital share	Habit persistence	Price markup	Wage markup	Price indexation
0.942	0.37	0.36	1.2	1.2	0.225
$\gamma_w$	$a^{oo}(1)$	$S^{oo}(1)$	$\delta_k$	$g$	$G_z$
Wage indexation	Utilization elasticity	Inv. Adjustment cost	Capital depreciation	Autonomous spending	TFP growth rate
0.295	2.246	5.16	0.015	1.6	0.256
Model Specific Parameters					
Expectations-trap			Secular stagnation		
$\delta$	$\phi_p$		$\delta$	$\phi_p$	
Mg. utility bonds	Price adj. cost		Mg. utility bonds	Price adj. cost	
0.0443	900		0.0454	1800	
$\phi_w$	$\omega$		$\phi_w$	$\omega$	
Wage adj. cost	Labor disutility		Wage adj. cost	Labor disutility	
115.5	0.62		92.7	0.59	

*Notes:* The parameter  $\beta$  is calibrated to obtain a positive natural rate in the presence of positive bond premium—see Section 2.4. The parameters  $\alpha, \gamma, \gamma_p, \gamma_w, a^{oo}(1), S^{oo}(1), \delta_k$  are taken from Hirose (2020, Table 2). The parameters  $\nu_p$  and  $\nu_w$  target 20 percent price and wage markups in the steady state. The parameters  $g$ , and  $G_z$  are set to match the average GDP share of autonomous spending  $(G + X - M)/Y$  and average per-capita GDP growth from 1998Q1 to 2012Q4, respectively. The model-specific parameters are calibrated as discussed in section 5.1.